CONTROL OF SWITCHED AFFINE SYSTEMS WITH BOUNDED SAMPLING TIME RATE ON THE SWITCHING FUNCTION: APPLICATION TO BUCK DC-DC CONVERTER

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Abstract **– The paper addresses the problem of designing a stabilizing control for switched affine and its experimental verification based on Linear Matrix Inequalities (LMIs). The main contribution is on the determination of a switching function that exploits the potential of LMI control approaches and which assures global stability and minimizes a guaranteed quadratic cost. Slack variables are introduced to reduce the design conservatism and new sufficient LMI conditions for the synthesis of the controllers are presented. Thus, it is showed that the performance of the control system is superior with a smaller guaranteed cost upper bound of that afforded by recent results. In addition, a theorem with sufficient conditions for the control of switched affine systems that allows a way to garantee a bounded sample time rate on the switching function is proposed. The implementation of the switching function taking into account a bounded sampling time is analysed and discussed with particular interest. Finally, the theoretical results are applied to Buck DC-DC converter. Several simulations show the usefulness of the methodology and experimental results obtained from a prototype validate this approach.**

Keywords **– Bounded Sampling Time, Buck DC-DC Converter, Linear Matrix Inequalities (LMIs), Lyapunov Function, Switched Affine System.**

I. INTRODUCTION

Switched systems are a subclass of hybrid systems and in the last years have witnessed the crescent study for the scientific community. This interest is due to the fact that the switched systems are applied in many important engineering areas, such as: control of mechanical systems, process control, power systems, aircraft control, automotive, power electronics, biomedical engineering, among others [1]–[7]. In general, the switched systems are characterized by having a switching rule that selects, at each instant of time, a dynamic subsystem among a determined number of available subsystems. However, special care should be taken regarding their mathematical analysis, because even if all their subsystems are stable, a system with an inadequate switching rule may present divergent paths. On the other hand, a suitable switching of unstable subsystems can generate stable trajectories. Thus, we conclude that the stability depends not only on the dynamics of the subsystems but also on the properties of the switching rule [8]. In general, the main objective of this control system is to establish a switching strategy that, given an equilibrium point, ensures the asymptotic stability, hence assuring adequate performance [1], [2], [9]. The techniques most used for this class of systems consist of choosing an appropriate Lyapunov function, for instance, a quadratic function [10] or piecewise quadratic [11], where the difference among them is the conservativeness in the conditions. The book [9] presents important results in this research. Among the application fields of switched systems this paper considers the power converters, that are widely used in industry [12]–[24]. The dynamics of DC-DC converters can be described by switched affine systems which include all nonlinearities of the system. However, in switched affine systems it is possible that the subsystems do not share a common equilibrium point and sometimes the stability concept should be extended using the ideas contained in [25].

Some results of primary works, about control applied in the field of power converters, can be found in [26] and [27] where the authors propose nonlinear strategies based on quadratic Lyapunov functions. More recently, [28]–[30] derive robust nonlinear controllers for power converters. Accordingly, in [17] the authors presented a state-dependent switching law for affine switching systems and its applications to LMI based design of some basic DC-DC converters, in order to reduce the losses due to the parasitic inductor resistance. In order to compare the performance indexes, two different laws for the basic DC-DC converters, namely the overshoot and the response time, are presented in [18]. The design and experimental validation of a Discrete Linear Quadratic Regulator (DLQR) are presented in [31]. A new theorem, whose conditions hold when the conditions of the two theorems proposed in [17] hold, is presented in [19]. Within context, in [20] some conditions were analyzed and a state-dependent switching strategy for switched affine systems was designed such that minimizes a quadratic guaranteed cost described in [17]. Additionally, in [21] the problem of establishing a state estimator for switched affine systems was studied and the proposed method relies on a simplification

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of estimation error, guaranteeing the estimation error to asymptotically converge to zero, for any initial state and switching law.

This work is concerned with the control methods design based on LMIs for a class of switched affine systems and these methods are applied to the control of a Buck DC-DC power converter. LMIs [32], when feasible, are easily solved by some tools available in the literature of convex programming [33]. In order to obtain less conservative conditions than afforded by recently results in the literature, a more general design procedure with performance indices, such as a decay rate (related to the setting time) and a guaranteed cost, were considered for this control system. In resume, the performance of the control system is superior with a maximum upper bound of the guaranteed cost smaller than afforded by recent results. This is an important contribution because the reduction of the conservatism with respect to previously published papers on the subject. However, the proposed control strategy designing is in general a discontinuous function. More specifically, the methods used to designing control strategy usually neglect the switching period and consequently the switching period becomes small enough, which can lead to a very high frequency and hence is usually not realizable in practice. In addition, high-frequency caused by the chattering are undesirable because they may excite high-frequency dynamics on the plant that were not modeled, which could result in unforeseen instabilities. This problem is very hard to solve. Owing to this interest, the other contribution of the paper is provide sufficient conditions for the control of switched affine systems that makes a know equilibrium point uniform ultimate boundedness [34]. Make the equilibrium point uniform ultimate boundedness is equivalent to say that the state variables of the system not necessarily converge to the origin of the system, but for a closed and bounded region around this origin. Then, our approach differs from previous works due allows to guarantee a bounded sampling time rate on the switching function. The last contribution of the paper has to do with the few experimental verifications of the proposed controller. The experimental measurements obtained are in agreement with the simulation and this validates the theory. Finally, a conclusion summarises the key aspects of the design method.

The notation used is described as follows. For real matrices or vectors $(')$ indicates transpose. The set composed by the first N positive integers, $1,\ldots,N$ is denoted by $\mathbb{K}$. The set of all vectors $\lambda = (\lambda_1,\ldots,\lambda_N)'$ such that $\lambda_i \geq 0$, $i = 1,2,\ldots,N$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_N = 1$ is denoted by Λ. The convex combination of a set of matrices $(A_1,\ldots,A_N)$ is denoted by $A_\lambda = \sum_{i=1}^{N} \lambda_i A_i$, where $\lambda \in \Lambda$. The trace of a matrix P is denoted by $Tr(P)$, $\mathbb{R}_+$ indicates the set of all positive real numbers and $sgn(\beta)$ denotes the signum function of β, which is equal to $1, 0$, or -1 if $\beta > 0$, $\beta = 0$, or $\beta < 0$, respectively.

II. SWITCHED AFFINE SYSTEMS

In this section, the problem to be dealt with is presented. The class of switched systems of interest is defined by the following state space realization [17]:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}w(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output, $w(t) \in \mathbb{R}^m$ is the input supposed to be constant for all $t \geq 0$ and $\sigma(t)\colon t \geq 0 \to \mathbb{K}$ is the switching strategy. For a known set of matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C_i \in \mathbb{R}^p$, $i \in \mathbb{K}$, such that $A_{\sigma(t)} \in \{A_1, A_2, \ldots, A_N\}$, $B_{\sigma(t)} \in \{B_1, B_2, \ldots, B_N\}$ and $C_{\sigma(t)} \in \{C_1, C_2, \ldots, C_N\}$, the switching strategy $\sigma(t)$ selects at each instant of time $t \geq 0$, a known subsystem among N available. The initial control problem in this paper is to determine a set of equilibrium point $x = x_e$ such that $\lim_{t\to\infty} x(t) = x_e$ holds for all initial condition $x_0 \in \mathbb{R}^n$ whenever the switched strategy is applied. Then, consider that the class of the dynamical systems (1), is also given by the convex combination of the N subsystems, which is described by:

$$\dot{x}(t) = A_\lambda x(t) + B_\lambda w(t). \tag{2}$$

Note that the control is now λ and it takes its values in the whole simplex Λ. An important feature of this class of dynamical systems is to describe the set of the equilibrium points of the switched affine systems. This fact is already noticed in [17], [25], [35]. The next definiton elucidates this characterization.

Definition 1. *Let X_e be the set of the equilibrium points related to system* (2) *and defined as:*

$$X_e := \left\{ x_e \in \mathbb{R}^n, \quad x_e = -A_\lambda^{-1} B_\lambda w(t), \quad \lambda \in \Lambda \right\}. \tag{3}$$

As it has been shown in [17], [25], [35], the state $x(t)$ of system (1) can be stabilized at any equilibrium point $x_e \in \mathbb{R}^n$ using the switched state feedback law, given by:

$$\sigma(x(t)) = arg \min_{i\in\mathbb{K}} (2(x-x_e)'P(A_i x + B_i w)), \tag{4}$$

where $P > 0$ satisfies the Lyapunov inequality:

$$A_\lambda' P + P A_\lambda < 0, \tag{5}$$

for some $\lambda \in \Lambda$. Another interest for the class of dynamical systems (1), is related to generalise this result in order to include performance indexes, for instance, a guaranteed cost and a lower bound for the decay rate. To be more precise, let a quadratic Lyapunov function $V(x-x_e) = (x-x_e)'P(x-x_e)$ and consider a quadratic guaranteed cost:

$$\min_{\sigma(t)\in\mathbb{K}} \int_0^\infty (z - H_{\sigma(t)}x_e)'(z - H_{\sigma(t)}x_e)dt$$
$$= \min_{\sigma(t)\in\mathbb{K}} \int_0^\infty (x-x_e)'Q_{\sigma(t)}(x-x_e)dt, \tag{6}$$

where $z = H_{\sigma(t)}x$, $Q_{\sigma(t)} = H_{\sigma(t)}' H_{\sigma(t)} \geq 0$ for $\sigma(t) \in \mathbb{K}$ and $x_e \in \mathbb{R}^n$ is a given equilibrium point. The next theorem provides conditions that allow the specification of the guaranteed cost and the decay rate in the same design.

Theorem 2. *[36] Consider the switched affine system* (1) *with a constant input $w(t) = w$ for all $t \geq 0$, let the equilibrium point $x_e \in \mathbb{R}^n$, $\gamma > 0$ be given and suppose that the state vector $x(t) \in \mathbb{R}^n$ is available for feedback. If there exist $\lambda \in \Lambda$, and a* *symmetric matrix $P \in \mathbb{R}^{n\times n}$, such that*

$$P > (2\gamma)^{-1}Q_i, \quad (7)$$
$$A_i'P + PA_i + 2\gamma P < 0, \quad (8)$$
$$A_\lambda x_e + B_\lambda w = 0, \quad (9)$$

where $Q_i = H_i'H_i$, $i \in \mathbb{K}$, then the switching strategy

$$\sigma(x(t)) = arg\min_{i\in\mathbb{K}}(2(x-x_e)'P(A_ix_e + B_iw)), \quad (10)$$

makes the equilibrium point $x_e \in \mathbb{R}^n$ of switched affine system (1) *globally exponentially stable with decay rate equal to or greater than γ and admits the guaranteed cost*

$$\mathscr{J} = \int_0^\infty (x-x_e)'Q_\sigma(x-x_e)dt < (x_0-x_e)'P(x_0-x_e). \quad (11)$$

Proof. See [36] for details. ■

Theorem 2 provides the following minimization problem:

$$\inf_{P>0}\{Tr(P) : (7)-(9) \quad \text{hold for some} \quad P = P' > 0 \quad \text{and} \quad i \in \mathbb{K}\}. \quad (12)$$

Moreover, note that the presented restriction (7) makes Theorem 2 conservative. In order to obtain less conservative conditions, we propose in the next theorem another way to consider the restrictions for decay rate and guaranteed cost.

Theorem 3. *[37] Consider the switched affine system* (1) *with constant input $w(t) = w$ for all $t \geq 0$, let the equilibrium point $x_e \in \mathbb{R}^n$, $\gamma > 0$ be given and suppose that the state vector $x(t) \in \mathbb{R}^n$ is available for feedback. If there exist $\lambda \in \Lambda$, symmetric matrices $Z_i \in \mathbb{R}^{n\times n}$ and a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$, such that*

$$Z_i > Q_i, \quad (13)$$
$$Z_i > 2\gamma P, \quad (14)$$
$$A_i'P + PA_i + Z_i < 0, \quad (15)$$
$$A_\lambda x_e + B_\lambda w = 0, \quad (16)$$

where $Q_i = H_i'H_i$, $i \in \mathbb{K}$, then the switching strategy (10) *makes the equilibrium point $x_e \in \mathbb{R}^n$ of switched affine system* (1) *globally exponentially stable with decay rate equal to or greater than γ and the guaranteed cost* (11) *holds.*

Proof. Consider the quadratic Lyapunov candidate function $V(x-x_e) = (x-x_e)'P(x-x_e)$. From (1), (10), (13), (15) and (16), one has for $(x-x_e) \neq 0$:

$$\begin{aligned}
\dot{V}(x-x_e) &= 2(x-x_e)'P(A_\sigma x + B_\sigma w)\\
&= 2(x-x_e)'P(A_\sigma x_e + B_\sigma w)\\
&\quad + (x-x_e)'(A_\sigma'P + PA_\sigma)(x-x_e)\\
&= \min_{i\in\mathbb{K}}\{2(x-x_e)'P(A_ix_e + B_iw)\}\\
&\quad + (x-x_e)'(A_\sigma'P + PA_\sigma)(x-x_e)\\
&< \min_{i\in\mathbb{K}}\{2(x-x_e)'P(A_ix_e + B_iw)\}\\
&\quad - (x-x_e)'Z_\sigma(x-x_e)\\
&\leq 2(x-x_e)'P(A_\lambda x_e + B_\lambda w) - (x-x_e)'Z_\sigma(x-x_e)\\
&= -(x-x_e)'Z_\sigma(x-x_e)\\
&< -(x-x_e)'Q_\sigma(x-x_e) \leq 0. \quad (17)
\end{aligned}$$

The second inequality in (17) is based on (10) and in the following fact: the minimum of a set of real numbers is smaller than or equal to any convex combination of these numbers [3]. Now, assuming that $(x-x_e) \neq 0$, then from (14) and (17), $\dot{V}(x-x_e) < -(x-x_e)'Z_\sigma(x-x_e) < -2\gamma(x-x_e)'P(x-x_e)$. Thus, the switched affine system has a decay rate equal to or greater than γ. Finally, integrating (17) from zero to infinity and taking into account that $V(x(\infty)-x_e) = 0$, it follows (11). The proof is concluded. ■

Theorem 3 provides the following minimization problem:

$$\inf_{P>0}\{Tr(P) : (13)-(16) \quad \text{hold for some} \quad P = P' > 0 \quad \text{and} \quad i \in \mathbb{K}\}. \quad (18)$$

The next theorem compares the conditions of Theorems 2 and 3.

Theorem 4. *[37] If the conditions of Theorem 2 hold, then the conditions of Theorem 3 also hold.*

Proof. Consider that (7)–(9) hold. Then, choose a constant β such that $\beta > 0$. Now, define $Z_i = 2\gamma P + \beta I_n$, $i \in \mathbb{K}$ and observe that the conditions (13)–(16) from Theorem 3 hold if the conditions (7)–(9) from Theorem 2 hold, for sufficiently small values of $\beta > 0$:

$$\begin{aligned}
&Z_i = 2\gamma P + \beta I_n > 2\gamma P > Q_i,\\
&Z_i = 2\gamma P + \beta I_n > 2\gamma P,\\
&A_i'P + PA_i' + Z_i = A_i'P + PA_i' + 2\gamma P + \beta I_n < 0, \quad (19)
\end{aligned}$$

and (9) is equal to (16). The proof is concluded. ■

The simulation results described in the next sections show that there exist cases where the conditions from Theorem 3 are less conservative than those proposed in Theorem 2.

Remark 5. *It is interesting to note that to solve the conditions related to Theorems 2 and 3 it is necessary to determine a specific vector $\lambda \in \Lambda$, which is associated with a known equilibrium point $x_e \in \mathbb{R}^n$. Then, for a given equilibrium point $x_e \in \mathbb{R}^n$, the associated vector $\lambda \in \Lambda$ is determined. However, a known problem in control systems is that the equilibrium point $x_e \in \mathbb{R}^n$ can vary over time. Thus, remembering that the solution of inequality* (16)*, related to Theorem 3, requires the determination of a specific value of $\lambda \in \Lambda$ that is associated with the equilibrium point $x_e \in \mathbb{R}^n$, then the proposed control strategy* (10) *does not guarantee global stability to a different equilibrium point than was projected. A possible solution to the proposed problem is to design a control strategy dependent only on partial equilibrium point information.*

III. BUCK DC-DC CONVERTER

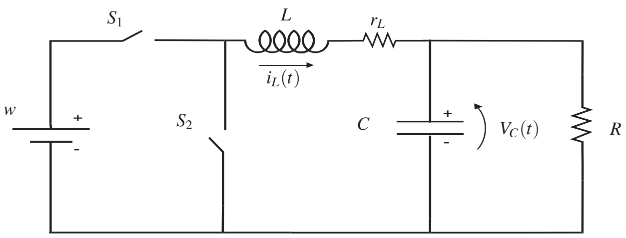

Fig. 1. Buck DC-DC converter.

Consider the Buck DC-DC converter shown in Figure 1. The converter is modeled as nonlinear switched affine system. Consider that $i_L(t)$ denotes the inductor current and $V_C(t)$ the capacitor voltage. Define the state vector $x(t)' = [x_1(t)\ x_2(t)] = [i_L(t)\ V_C(t)]$ and consider an equilibrium point $x_e' = [x_{1e}\ x_{2e}] = [i_{Le}\ \ V_{Ce}]$. Now, assume that the systems described by (1) consist of a group of $N = 2$ affine subsystems sharing the same state vector and the decision of which subsystem is active is the control variable, resulting in a switching strategy $\sigma(x - x_e) \in \{1, 2\}$. Initially, for theoretical analysis of the Buck DC-DC converter, no limit is imposed on the switching frequency because the trajectory of the system evolves on a sliding surface with infinite frequency. Then, it is not difficult to verify that this system is given as [17], [19]:

$$A_1 = \begin{bmatrix} -r_L/L & -1/L \\ 1/C & -1/RC \end{bmatrix},\ A_2 = \begin{bmatrix} -r_L/L & -1/L \\ 1/C & -1/RC \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{20}$$

Now, for the analysis of Theorems 2 and 3, suppose the following values to the parameters:

$$w = 24V,\ R = 15\Omega,\ r_L = 2.6\Omega,\ L = 3.6\ mH,\ C = 10\mu F, \tag{21}$$

and $Q_i = diag\{\rho_1 r_L,\ \rho_2/R\}$, where $Q_i \geq 0,\ i \in \mathbb{K}$, is the performance index matrix associated with the guaranteed cost:

$$\int_0^\infty (\rho_2 R^{-1}(V_C - V_{Ce})^2 + \rho_1 r_L (i_L - i_{Le})^2 dt, \tag{22}$$

which ρ_1 and $\rho_2 \in \mathbb{R}_+$ are design parameters that plays an important role with regard to the peak current value and settling time of the voltage [17]. In this study, consider $\rho_1 = 0$ and $\rho_2 = 1$. The set of all attainable equilibrium points of the Buck DC-DC converter is given by [17]:

$$x_e = \left\{ [i_{Le}\quad V_{Ce}]' : V_{Ce} = R i_{Le},\quad 0 \leq i_{Le} \leq \frac{w}{(r_L + R)} \right\}. \tag{23}$$

For the analysis of the Buck DC-DC converter, suppose the following value of load voltage $V_{Ce} = 6V$ and a decay rate $\gamma = 42s^{-1}$. From (23) the equilibrium point is $x_e' = [i_{Le}\quad V_{Ce}] = [0.4\quad 6]$. Therefore, from (9) one obtains $\lambda_1 = 0.2933$ and $\lambda_2 = 0.7067$. Moreover, observe that the proposed design method can also be used for different values of the parameters or reference points. Then, from the minimization problem (12), corresponding to Theorem 2, the solution was the following matrix:

$$P = \begin{bmatrix} 0.0911 & -0.0027 \\ -0.0027 & 0.0009 \end{bmatrix}. \tag{24}$$

Considering the same parameters above, from (16) one obtains $\lambda_1 = 0.2933$ and $\lambda_2 = 0.7067$. Then, from the minimization problem (18), related to Theorem 3, the obtained solution was the following matrix:

$$P = 10^{-4} \times \begin{bmatrix} 13.9213 & 0.0946 \\ 0.0946 & 0.0464 \end{bmatrix}. \tag{25}$$

Simulation results for the initial conditions $(x_1(0), x_2(0))' = (0, 0)$ and $(1, 15)$, considering the proposed method and the procedure presented in [36], described in Theorem 2, are shown in Figures 2 to 4.

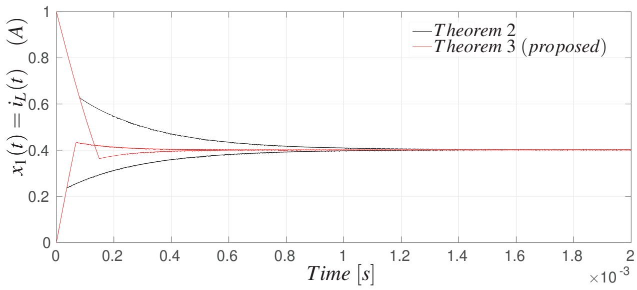

Fig. 2. Time simulations of the state variable $x_1(t) = i_L(t)$ (the reference value is $x_{1e} = 0.4\ A$).

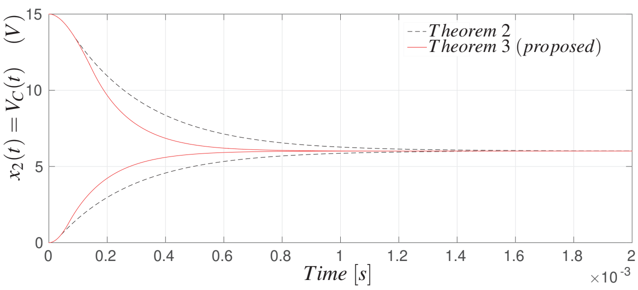

Fig. 3. Time simulations of the state variable $x_2(t) = V_C(t)$ (the reference value is $x_{2e} = 6\ V$).

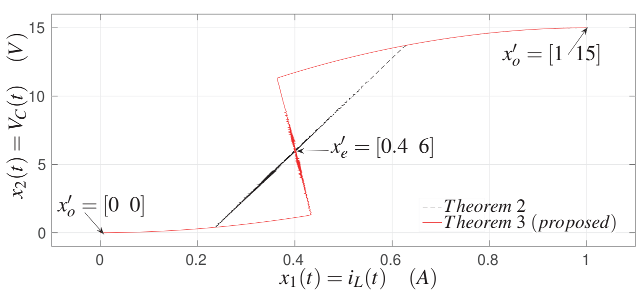

Fig. 4. Phase plane of the state variables $x_1(t) = i_L(t)$ and $x_2(t) = V_C(t)$.

Observe that the proposed method presents a better convergence speed. Table I displays the obtained results.

TABLE I
Buck DC-DC Converter Results for $\gamma = 42$

	Initial conditions	Voltage settling time	Guaranteed cost (11)
Theorem 2	$(0,0)$	1.00 *ms*	0.03310
Theorem 3	$(0,0)$	0.50 *ms*	0.00043
Theorem 2	$(1,15)$	1.25 *ms*	0.07450
Theorem 3	$(1,15)$	0.75 *ms*	0.00097

Moreover, from Table I also note that the proposed method presents a smaller cost when compared with the cost obtained with Theorem 2. This fact illustrates the results presented in Theorem 4.

Nevertheless, a key assumption behind the switching function (10) is the non-continuous nature. Note that the controller proposed operates with a high switching frequency and this fact makes it impossible to be implemented in practical systems. This ideal switching phenomenon is known as chattering. Furthermore, high-frequency components caused by the chattering are undesirable, because they may excite high-frequency dynamics on the plant that were not modeled, which could result in unforeseen instabilities. This problem is very hard to solve. Owing to this interest, we provide in the next theorem sufficient conditions for the control of switched affine systems that allows a way to guarantee a bounded sampling time rate on the switching function. The next theorem provides an important result on this subject.

Theorem 6. *[37] Consider the switched affine system* (1) *with constant input $w(t) = w$ for all $t \geq 0$. If the LMIs* (13)−(16) *of Theorem 3 hold, then the switching strategy* (10)*, with $\sigma(t) = \sigma(kT)$, $kT \leq t < (k+1)T$, $k = 0,1,2,\ldots$, for all constant $T > 0$, makes the equilibrium point $x_e \in \mathbb{R}^n$ for the switched affine system* (1) *uniformly ultimate bounded.*

Proof. Consider the quadratic Lyapunov function candidate $V(x-x_e) = (x-x_e)'P(x-x_e)$ where $x_e \in \mathbb{R}^n$ is a know equilibrium point. From (1), (10), (14), (15) and $(x-x_e) \neq 0$, one has:

$$\begin{aligned}
\dot{V}(x-x_e) &= 2(x-x_e)'P(A_\sigma x + B_\sigma w) \\
&= 2(x-x_e)'P(A_\sigma x_e + B_\sigma w) \\
&\quad + (x-x_e)'(A_\sigma' P + PA_\sigma)(x-x_e) \\
&= 2(x-x_e)'P(A_\sigma x_e + B_\sigma w) \\
&\quad + (x-x_e)'(A_\sigma' P + PA_\sigma + Z_\sigma)(x-x_e) \\
&\quad - (x-x_e)'Z_\sigma(x-x_e) \\
&< 2(x-x_e)'P(A_\sigma x_e + B_\sigma w) - (x-x_e)'Z_\sigma(x-x_e) \\
&= \min_{i \in \mathbb{K}} \left(2(x-x_e)'P(A_i x_e + B_i w)\right) \\
&\quad - (x-x_e)'Z_\sigma(x-x_e) \\
&< \min_{i \in \mathbb{K}} \left(2(x-x_e)'P(A_i x_e + B_i w)\right) \\
&\quad - (x-x_e)'2\gamma P(x-x_e) \\
&\leq -\varepsilon_1 \left\|(x-x_e)\right\|^2 + \varepsilon_2 \left\|(x-x_e)\right\|,
\end{aligned} \tag{26}$$

where $-\varepsilon_1 < 0$ denotes the maximum eigenvalue of $-(2\gamma P)$, $\gamma > 0$, $P > 0$ and $\varepsilon_2 > 0$ represents the maximum value of $\|2P(A_i x_e + B_i w)\|$, $i \in \mathbb{K}$ where $\|(x-x_e)\| = \sqrt{(x-x_e)'(x-x_e)}$. Thus, for $(x-x_e) \neq 0$, $\dot{V}(x-x_e) < 0$ if $||(x-x_e)|| > \varepsilon_2/\varepsilon_1$ and according to [34] the controlled system is uniformly ultimately bounded. Moreover, assuming that $(x-x_e) \neq 0$, then from (13), (26) and recalling that $Q_i \geq 0$, $i \in \mathbb{K}$, then $\dot{V}(x-x_e) < -(x-x_e)'Z_\sigma(x-x_e) < -(x-x_e)'Q_\sigma(x-x_e) \leq 0$. The proof is concluded. ■

Remark 7. *Say that the switching strategy $\sigma(t) = \sigma(kT)$, for $kT \leq t \leq (k+1)T$, $k = 0,1,2,\ldots$ makes the switched affine system uniform ultimate bounded is equivalent to say that the state variables of the system not necessarily converge to the origin of the system, but for a closed and bounded region around this origin. Furthermore, this bounded region becomes smaller when the sampling time rate (T) on the switching function $\sigma(t) = \sigma(kT)$ decreases.*

Remark 8. *From* (11)*, observe that for $t \to \infty$ the guaranteed cost of the system, controlled by the switching strategy $\sigma(t) = \sigma(kT)$, $kT \leq t < (k+1)T$ and $k = 0,1,2,\ldots$, is infinity. Thus, the guaranteed cost of the controlled system can be interpreted as the necessary cost for the controlled system converge to a neighborhood of the equilibrium point.*

In the next section, this procedure is applied to the Buck DC-DC converter in order to illustrate the control design method developed.

IV. BUCK DC-DC CONVERTER OPERATING WITH BOUNDED SAMPLING TIME RATE

Consider the same parameters previously defined in (21). Adopting the following value of load voltage $V_{Ce} = 6V$ and a maximum decay rate $\gamma = 42s^{-1}$. From (23) the reference point is $x_e = [i_{Le} \quad V_{Ce}]' = [0.4 \quad 6]'$. Then, from Theorem 6, note that the minimization problem is the same to (18), related to Theorem 3. Thus, the obtained solution was the same matrix (25). Simulation results for the initial condition $(x_1(0),\ x_2(0)) = (0,0)$, obtained using a switching strategy $\sigma(t) = \sigma(kT)$, $kT \leq t < (k+1)T$, $k = 0,1,2,\ldots$, proposed by Theorem 6, and a bounded sampling time rate equal to $T = 10\mu s$ are shown in Figures 5 to 8.

From Figures 5 and 7, note that even with a bounded sampling time rate, the proposed switching strategy $\sigma(t) = \sigma(kT)$, $kT \leq t < (k+1)T$ and $k = 0,1,2,\ldots$, leads the state variables to a bounded region around the desired equilibrium point. Moreover, from Figure 6, observe that the voltage settling time is equal to that obtained when no limit is imposed on the sampling time rate, which highlights the quality of the results. Table II presents the obtained results.

TABLE II
Buck DC-DC Converter Results

	Steady state $x_2(t) = V_c(t)$	Voltage settling time	Sampling time rate
Theorem 3	6.00 *V*	0.50 *ms*	*no limit*
Theorem 6	6.17 *V*	0.50 *ms*	$10\mu s$

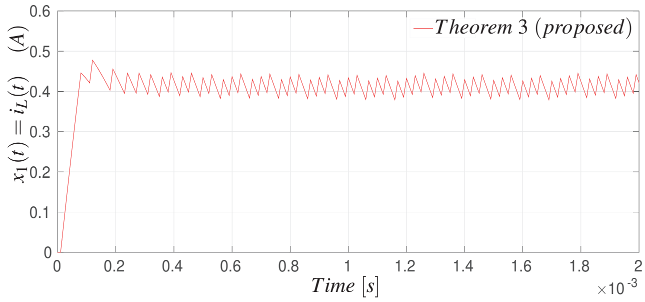

Fig. 5. Time simulations of the state variable $x_1(t) = i_L(t)$ (the reference value is $x_{1e} = 0.4\,A$).

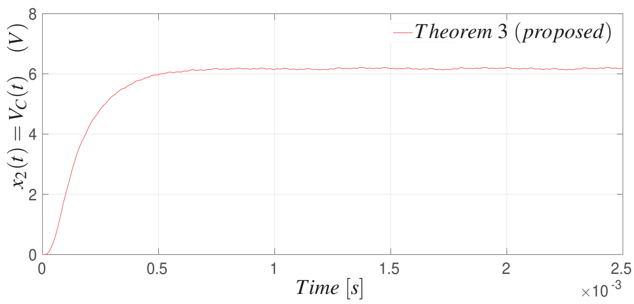

Fig. 6. Time simulations of the state variable $x_2(t) = V_C(t)$ (the reference value is $x_{2e} = 6\,V$).

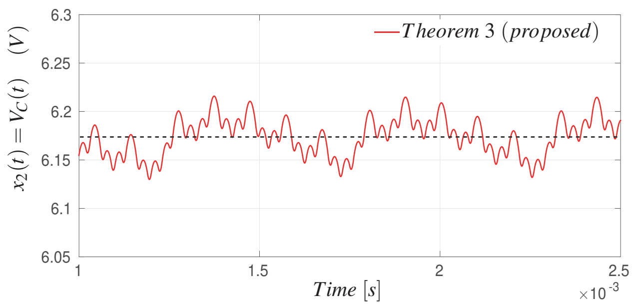

Fig. 7. Amplification of the state variable $x_2(t) = V_C(t)$ (the reference value is $x_{2e} = 6\,V$).

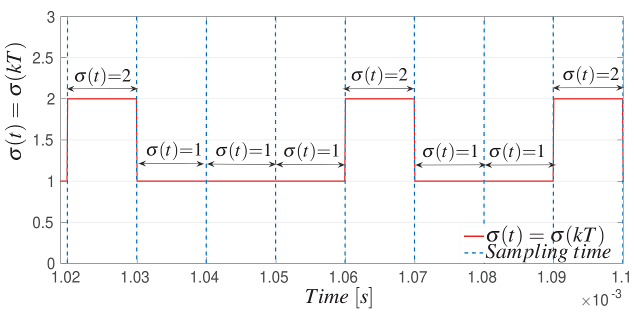

Fig. 8. Time simulations of the switching strategy $\sigma(t) = \sigma(kT)$, $kT \leq t < (k+1)T, k = 0,1,2,\ldots$

V. EXPERIMENTAL VERIFICATION FOR BUCK DC-DC CONVERTER OPERATING WITH BOUNDED SAMPLING TIME RATE

To demonstrate the advantages of the proposed control, an experimental prototype of a Buck DC-DC converter has been implemented. The structure of the converter with the controller is shown in Figure 9 and an electronic configuration for the DC-DC converter is shown in Figure 10. The design parameters are the same defined previously in (21).

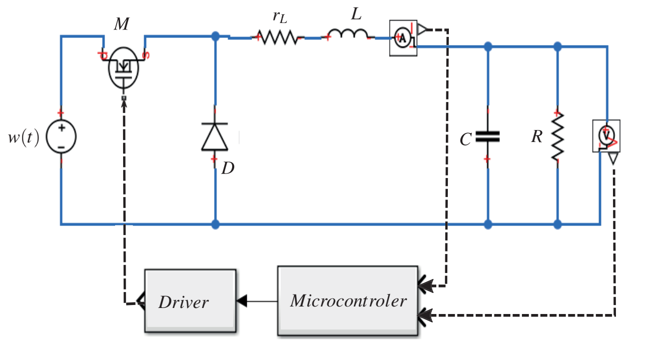

Fig. 9. Equivalent electric scheme for control of Buck DC-DC converter.

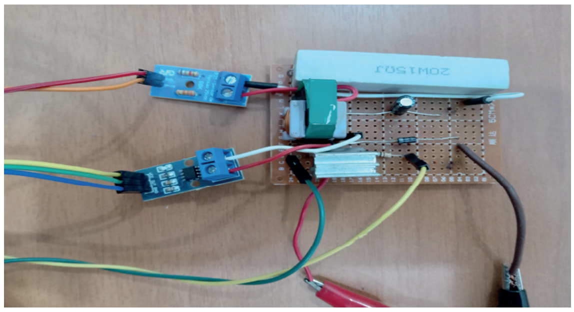

Fig. 10. Electric scheme for control of Buck DC-DC converter.

Note that the switch S_1 was changed by the MOSFET transistor M and the control of $\sigma(t)$ on M is the following: $\sigma(t) = 1$ when M is ON and $\sigma(t) = 2$ when M is OFF. Then, from the switching strategy (10), $i \in \{1,2\}$, define the following signal: $\mathscr{H} = 2(x-x_e)'P((A_1-A_2)x_e+(B_1-B_2)w)$. Next, from (20), (21), (25) and recalling that the state vector $x(t)' = [x_1(t)\; x_2(t)] = [i_L(t)\; V_C(t)]$ and the equilibrium point $x_e' = [x_{1e}\; x_{2e}] = [i_{Le}\; V_{Ce}]$, one has:

$$\mathscr{H} = 18.5644(i_L(t) - i_{Le}) + 0.1261(V_C(t) - V_{Ce}). \qquad (27)$$

Again, recalling that $sgn(\mathscr{H})$ denotes the signum function of $\mathscr{H}$, which is equal to $1, 0$, or -1 if $\mathscr{H} > 0$, $\mathscr{H} = 0$ or $\mathscr{H} < 0$, respectively, one has, from (10), the following switching rule described in Table III.

TABLE III
Switching Rule

	$sgn(\mathscr{H})$	$\sigma(t)$	S_1	$MOSFET(M)$
$if\ \mathscr{H} < 0$	-1	1	1	ON
$if\ \mathscr{H} > 0$	1	2	0	OFF

Now, from Table III, the switching strategy $\sigma(t)$ can be implemented by a microcontroller with periodic interrupts. At each interruption, the controller reads the state variables values through the sensors signals received from the A/D converter and makes the decision of which subsystem should be activated. Therefore, note that the switching frequency is not infinite as assumed in the theoretical developments. This fact keeps the MOSFET operating with a bounded sampling time rate and its illustrates the results presented in Theorem 6. From the aforementioned analysis, in order to demonstrate the effectiveness of the proposed method, assume that the sampling time rate is $T = 10\mu s$. Then, Figure 11 depicts the transient digital response for the current $i_L(t)$ and Figure 12 shows the transient response for the voltage signal $V_C(t)$, obtained when the converter start-up and operating at the nominal equilibrium point.

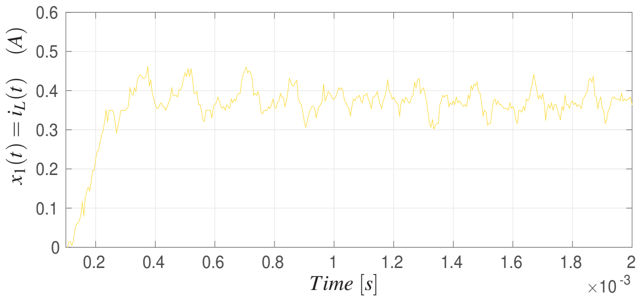

Fig. 11. Digital response of the state variable $x_1(t) = i_L(t)$ (the reference value is $x_{1e} = 0.4\ A$).

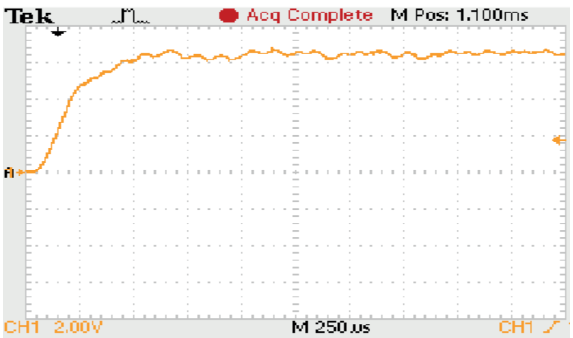

Fig. 12. Transient response for the state variable $x_2(t) = V_C(t)$ (the reference value is $x_{2e} = 6\ V$).

From Figure 11, note that the amplitude and frequency of the current $iL(t)$ diverges from the result illustrated in Figure 5. This fact is due to the sensing and analog conditioning of the current signal. More specifically, the current at the equilibrium point is equal to $0.4A$ and the sensor used $(ACS712)$ has a nominal current equal to $3A$, then concludes that the sensor operates with an analog gain smaller than adequate. Moreover, from Figure 12, observe that the voltage settling time obtained is $5ms$ and it is equal to that obtained in Figure 6.

VI. COMPARATION OF THE BUCK DC-DC CONVERTER OPERATING WITH PI AND PI WITH LEAD CONTROLLER

In this example, the main idea is to compare the proposed results by the Theorem 6 with the classics controllers in the literature, more specifically PI and PI with Lead controller. Then, for the DC-DC Buck converter illustrated in Figure 1, consider the same parameters previously defined in (21) and the following value of load voltage $V_{Ce} = 6V$. Results of Frequency Domain analysis of the Phase Margin and the Crossover Frequency is tabulated in Table IV.

TABLE IV
Frequency Domain Analysis for PI and PI with Lead Compensation

Controller	Phase Margin in Degrees	Gain cross over Frequency (kHz)
PI	30.0	2.0
PI with Lead	50.0	5.0

Then, for the frequency domain analysis presented in Table IV, one has for PI compensator, $k_p = 0.234$ and $k_i = 294.294$, where K_p and K_i are the parameters of PI controller. Following, for PI with Lead controller, one has, $k_p = 1.473$, $k_i = 4626.790$, $G_{lead} = 0.444$, $w_z = -13954.430$ and $w_p = -70727.370$, where K_p and K_i are the parameters of PI controller and G_{lead}, w_z and w_p are the parameters of lead compensator. Simulation results for the initial condition $(x_1(0),\ x_2(0)) = (0,0)$, using a switching strategy $\sigma(t) = \sigma(kT)$, $kT \leq t < (k+1)T$, $k = 0,1,2,\ldots$, proposed by Theorem 6, PI and PI with Lead controllers are shown in Figures 13 and 14.

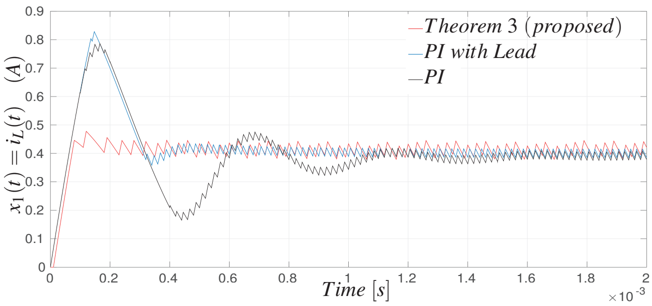

Fig. 13. Time simulations of the state variable $x_1(t) = i_L(t)$ (the reference value is $x_{1e} = 0.4\ A$).

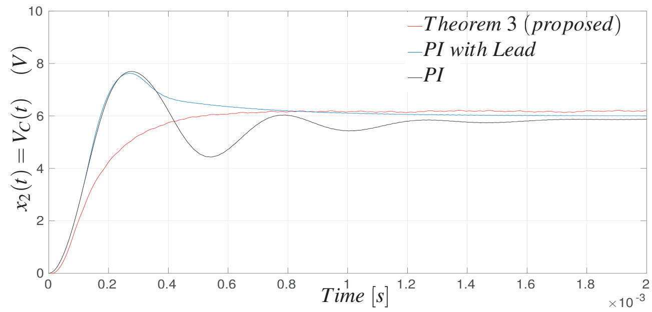

Fig. 14. Time simulations of the state variable $x_2(t) = V_C(t)$ (the reference value is $x_{2e} = 6\ V$).

The comparative results are presented in Table V.

TABLE V
Analysis for DC-DC Buck Converter

	Steady state $x_2(t) = V_c(t)$	Voltage settling time	Voltage Overshoot
Theorem 6	6.17 V	0.50 ms	0 V
PI with Lead	6.09 V	1.00 ms	7.80 V
PI	5,85 V	1.60 ms	7.80 V

The time domain response shows that the PI and the PI with lead compensation has a large settling time when compared to the proposed control technique. More specifically, the proposed control technique gives a good steady state and transient response. response

VII. CONCLUSION

In this paper, we have addressed a study and design for a new switching strategy for switched affine systems. The control design was based on Lyapunov stability using a quadratic function and LMIs. A quadratic guaranteed cost has been minimised. The main advantage of this approach is that the proposed design method can improve the convergence speed of a Buck DC-DC power converter, with a smaller quadratic guaranteed cost. Also, we have proposed a new control strategy in order to guarantee uniform ultimate boundedness for switched affine systems that allows to guarantee a bounded sampling time on the switching function. With the proposed method, steady-state performances can be taken into account and the resulting design has good performance characteristics despite the conservatism in the sampling time model. Besides, the experimental measurements from a prototype of the Buck power converter have verified the results and it showed a great agreement with the design. Finally, the approach used here can be extended to other more complex converters such as Boost, Buck-Boost and Sepic.

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