THE CHARGING AND DISCHARGING OF REVERSE BIASED SCHOTTKY DIODE VOLTAGE-DEPENDENT CAPACITOR

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***Abstract* – Transient analysis of RC circuit with voltage step source, constant resistor and nonlinear voltage-dependent capacitors with the capacitance given by $C(v_c) = C_o \big/ \sqrt{1 + v_c / V_{bi}}$, where C_o and V_{bi} are constant and v_c is the voltage across the capacitor, was carried out. The analysis was restricted to this class of nonlinear capacitors, because they represent with acceptable approximation the capacitances of reverse-biased Schottky diodes, which are very important and usually used in several static power converters practical applications. The charging and discharging are described by explicitly solvable nonlinear first-order differential equations. The theoretical analysis results are verified by numerical examples with computer simulation and also by experimentation in the laboratory.**

***Keywords* – RC circuit, transient analysis, voltage-dependent capacitor, Schottky diode.**

I. INTRODUCTION

It is well known that a Schottky diode biased in the reverse direction exhibits a voltage-variable capacitance and that this characteristic can be useful in a number of applications, which includes voltage tuning of an LC resonant circuit, voltage-controlled oscillators, parametric amplifiers and frequency multipliers.

Nonlinear voltage-dependent capacitances are also inherently present in silicon, GaN and SiC power semiconductor devices, where they can have a significant impact on the operation of high frequency switched-mode power converters. They can affect the switching times, commutation losses and the converter dynamics and can limit the maximum operation frequency of these converters.

In [1] the study on the charging and discharging of two types of nonlinear semiconductor capacitors through linear resistor is reported: one having a capacitance increasing exponentially with the voltage across it and the other, the space-charge capacitor that obey the function $C(v_c) = C_o \sinh(\alpha v_c) \big/ \alpha v_c$. In [2] a solution for the large signal transient response of a reverse biased p-n junction diode to a step voltage is presented, but only during the charging period. Reference [3] presents an analysis of the large signal step response of a junction capacitor considering charging and discharging through a linear resistance. It is shown that the behavior of an abrupt or a linearly graded junction capacitor is different from that of a conventional and voltage-independent capacitor. In [4], using a closed-form solution of the static Poisson's equation in the depletion layer, the large signal step response of a single-diffused junction is studied and it is shown that it deviates from that of either a linearly graded or abrupt junction. However, none of these articles present an explicit solution to the nonlinear differential equations that describe the transient behavior of the voltage across the capacitors.

Reference [5] has shown how the optimal solution for charging linear and nonlinear capacitor is computed, including the optimal charging voltage for a MOSFET gate capacitor, which capacitance is given by $C(v_c) = C_o \big/ \sqrt{1 + v_c / V_{bi}}$. Reference [6] also analyzes RC circuits containing a voltage-dependent capacitor, expanding and generalizing the analysis presented in [5].

However, as the objective of these studies was to find how the input voltage should change with respect to time to minimize the resistor losses, no explicit expressions for the voltage across the capacitor as a function of time was provided, when the circuit is supplied by an input voltage step.

In the research reported herein we investigate the transient behavior of RC networks containing nonlinear voltage-dependent capacitors. The analysis was restricted to basic circuits containing capacitors given by $C(v_c) = C_o \big/ \sqrt{1 + v_c / V_{bi}}$, which represents the reverse biased Schottky diode voltage-dependent capacitance. With this class of capacitors, as demonstrated in this paper, it is possible to find a closed form solution for the voltage across the nonlinear capacitors, the energy converted into heat in the series resistor, the energy stored in the capacitor, and the charging and discharging times.

II. THE SCHOTTKY DIODE VOLTAGE-DEPENDENT CAPACITANCE

Let the voltage-dependent capacitor be that shown in Figure 1.

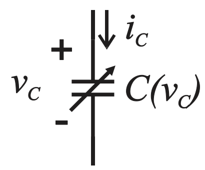

Fig. 1. Voltage-dependent capacitor.

The current flowing through the capacitor is given by

Manuscript received 07/24/2023; first revision 08/28/2023; accepted for publication 10/10/2023, by recommendation of Associate Editor Heverton Pereira. http://dx.doi.org/10.18618/REP.2023.4.0019

$$i_C = \frac{dQ(v_C)}{dt} \tag{1}$$

where denotes the electric charge on the capacitor plates, which is given by

$$Q(v_C) = C(v_C)v_C. \tag{2}$$

We can rewrite (1) as

$$i_C = \frac{dQ(v_C)}{dv_C}\frac{dv_C}{dt} \tag{3}$$

which substituted in (3) yields [7]

$$i_C = C(v_C)\frac{dv_C}{dt}. \tag{4}$$

The voltage-dependent depletion-layer capacitance of a reverse-biased Schottky diode is given by [8]

$$C(v_c) = C_o \Big/ \sqrt{1 + v_c / V_{bi}} \tag{5}$$

where C_o is the zero-bias junction capacitance and V_{bi} is the built-in voltage of the Schottky diode.

The measured $C-V$ characteristics of the Schottky diode STPS 10L25 from STMicroelectronics are shown in Figure 2 [7].

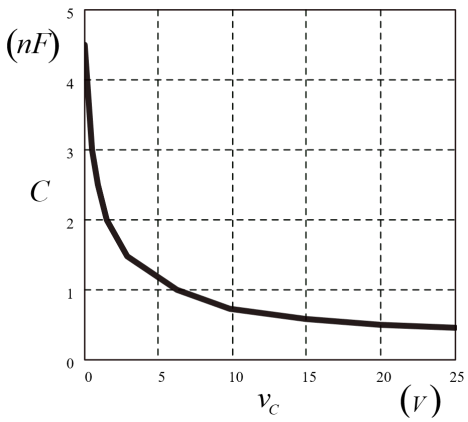

Fig. 2. Measured C-V characteristics of STPS10L25 [7].

From the characteristics shown in Fig. 2, we can extract the parameters of equation (6), which $C_o = 4.5\,nF$ are and $V_{bi} = 0.326\,V$.

III. CHARGING TRANSIENT OF THE VOLTAGE-DEPENDENT CAPACITOR

Let us consider the circuit shown in Figure 3. At $t = 0$ the voltage source V_1 is connected and the capacitor, with no initial voltage, begins to charge.

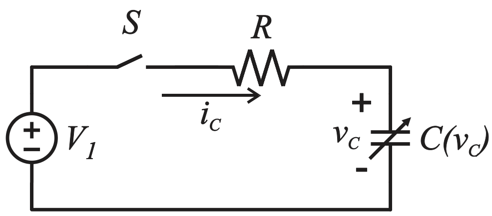

Fig. 3. Circuit for charging the voltage-dependent capacitor through the linear resistor R.

As follows from Kirchhoff's voltage law

$$V_1 = Ri_C + v_C. \tag{6}$$

Since

$$i_C = C(v_C)\frac{dv_C}{dt} \tag{7}$$

we have

$$C(v_c)\frac{dv_C}{dt} = \frac{V_1}{R} - \frac{v_C}{R}. \tag{8}$$

Substitution of (5) in (8) yields

$$C(v_c)\frac{dv_C}{dt} = \frac{V_1}{R} - \frac{v_C}{R}. \tag{9}$$

Thus

$$\frac{dv_C}{dt} = \frac{(V_1 - v_c)\sqrt{V_{bi} + v_c}}{RC_o\sqrt{V_{bi}}}. \tag{10}$$

From (10) we obtain

$$\frac{dv_C}{(V_1 - v_c)\sqrt{V_{bi} + v_c}} = \frac{dt}{RC_o\sqrt{V_{bi}}}. \tag{11}$$

Therefore,

$$\int \frac{dv_C}{(V_1 - v_c)\sqrt{V_{bi} + v_c}} = \int \frac{dt}{RC_o\sqrt{V_{bi}}} + K \tag{12}$$

where K is the constant of integration.

It is well known that

$$\int \frac{dt}{RC_o\sqrt{V_{bi}}} = \frac{t}{RC_o\sqrt{V_{bi}}}. \tag{13}$$

Integrating the first term of (12) we find

$$\int \frac{dv_C}{\left(V_1 - v_c\right)\sqrt{V_{bi} + v_c}} = \frac{1}{\sqrt{V_{bi} + V_1}} \left[\begin{array}{l} \ln\left(\dfrac{1}{\sqrt{V_{bi} + v_c}} + \dfrac{1}{\sqrt{V_{bi} + V_1}} \right) \\ -\ln\left(\dfrac{1}{\sqrt{V_{bi} + v_c}} - \dfrac{1}{\sqrt{V_{bi} + V_1}} \right) \end{array} \right]. \quad (14)$$

After appropriate algebraic manipulation we obtain

$$\int \frac{dv_C}{\left(V_1 - v_c\right)\sqrt{V_{bi} + v_c}} = \frac{1}{\sqrt{V_{bi} + V_1}} \ln\left(\frac{\dfrac{1}{\sqrt{V_{bi} + v_c}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}{\dfrac{1}{\sqrt{V_{bi} + v_c}} - \dfrac{1}{\sqrt{V_{bi} + V_1}}} \right). \quad (15)$$

In order to determine K, we substitute (14) and (15) in (13) and set $t = 0$ and $v_c\left(0\right) = 0$. Hence,

$$K = \frac{1}{\sqrt{V_{bi} + V_1}} \ln\left(\frac{\dfrac{1}{\sqrt{V_{bi}}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}{\dfrac{1}{\sqrt{V_{bi}}} - \dfrac{1}{\sqrt{V_{bi} + V_1}}} \right). \quad (16)$$

Substitution of (13), (15) and (16) in (12) yields

$$\begin{array}{l} \dfrac{1}{\sqrt{V_{bi} + V_1}} \left[\begin{array}{l} \ln\left(\dfrac{1}{\sqrt{V_{bi} + v_c}} + \dfrac{1}{\sqrt{V_{bi} + V_1}} \right) \\ -\ln\left(\dfrac{1}{\sqrt{V_{bi} + v_c}} - \dfrac{1}{\sqrt{V_{bi} + V_1}} \right) \end{array} \right] = \\ \dfrac{1}{\sqrt{V_{bi} + V_1}} \left[\begin{array}{l} \ln\left(\dfrac{1}{\sqrt{V_{bi}}} + \dfrac{1}{\sqrt{V_{bi} + V_1}} \right) \\ -\ln\left(\dfrac{1}{\sqrt{V_{bi}}} - \dfrac{1}{\sqrt{V_{bi} + V_1}} \right) \end{array} \right] + \dfrac{t}{RC_o\sqrt{V_{bi}}}. \end{array} \quad (17)$$

Rearranging the terms of (17) we find

$$\frac{t\sqrt{V_{bi} + V_1}}{RC_o\sqrt{V_{bi}}} = \ln\left(\frac{\dfrac{\dfrac{1}{\sqrt{V_{bi} + v_c}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}{\dfrac{1}{\sqrt{V_{bi} + v_c}} - \dfrac{1}{\sqrt{V_{bi} + V_1}}}}{\dfrac{\dfrac{1}{\sqrt{V_{bi} + v_c}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}{-\dfrac{1}{\sqrt{V_{bi}}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}} \right). \quad (18)$$

Let α be defined by

$$\alpha = \frac{\sqrt{V_1 + V_{bi}}}{RC_o\sqrt{V_{bi}}}. \quad (19)$$

Substituting (19) in (18) we find

$$\frac{\dfrac{1}{\sqrt{V_{bi} + v_c}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}{\dfrac{1}{\sqrt{V_{bi} + v_c}} - \dfrac{1}{\sqrt{V_{bi} + V_1}}} = e^{\alpha t} \left(\frac{\dfrac{1}{\sqrt{V_{bi}}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}}{-\dfrac{1}{\sqrt{V_{bi}}} + \dfrac{1}{\sqrt{V_{bi} + V_1}}} \right). \quad (20)$$

Solving (20) for $\sqrt{v_c + V_{bi}}$ we obtain

$$\sqrt{V_{bi} + v_c} = \frac{\begin{array}{c} \sqrt{V_{bi}}\sqrt{V_{bi} + V_1} - V_{bi} - V_1 + V_1 e^{\alpha t} \\ + V_{bi} e^{\alpha t} + \sqrt{V_{bi}}\sqrt{V_{bi} + V_1} e^{\alpha t} \end{array}}{\sqrt{V_{bi}} e^{\alpha t} + \sqrt{V_{bi} + V_1} - \sqrt{V_{bi}} + \sqrt{V_{bi} + V_1} e^{\alpha t}}. \quad (21)$$

From (21) we find

$$v_c = \left(\frac{\begin{array}{c} \sqrt{V_{bi}}\sqrt{V_{bi} + V_1} - V_{bi} - V_1 + V_1 e^{\alpha t} \\ + V_{bi} e^{\alpha t} + \sqrt{V_{bi}}\sqrt{V_{bi} + V_1} e^{\alpha t} \end{array}}{\sqrt{V_{bi}} e^{\alpha t} + \sqrt{V_{bi} + V_1} - \sqrt{V_{bi}} + \sqrt{V_{bi} + V_1} e^{\alpha t}} \right)^2 - V_{bi}. \quad (22)$$

Equation (22) represents the behavior of the voltage across the voltage-dependent capacitor of the electric circuit shown in Figure. 3.

Let γ, $\overline{v_c}$ and $\bar{t}$ be defined by (23), (24) and (25), respectively.

$$\gamma = \frac{V_{bi}}{V_1}. \quad (23)$$

$$\overline{v_c} = \frac{v_c}{V_1} \quad (24)$$

and

$$\bar{t} = \frac{t}{RC_o}. \quad (25)$$

Substituting (23), (24) and (25) in (22) and rearranging the terms we obtain

$$\overline{v_c} = \left(\frac{\left(\sqrt{\left(1+\gamma\right)\gamma} - \gamma - 1\right) + \left(\sqrt{\left(1+\gamma\right)\gamma} + \gamma + 1\right) e^{\bar{t}\sqrt{1+\gamma/\gamma}}}{\left(\sqrt{\left(1+\gamma\right)} - \sqrt{\gamma}\right) + \left(\sqrt{\left(1+\gamma\right)} + \sqrt{\gamma}\right) e^{\bar{t}\sqrt{1+\gamma/\gamma}}} \right)^2 - \gamma. \quad (26)$$

Figure 4 shows the plot of the normalized voltage $\overline{v_c}$ across the voltage-dependent capacitor of the circuit shown in Figure. 3, as function of the normalized time $\bar{t}$, for different values of γ.

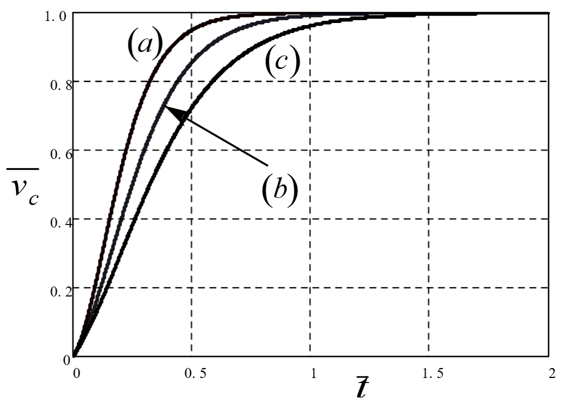

Fig.4. Normalized transient voltage $\overline{v_c}$ across the voltage-dependent capacitor in Figure 3 as function of the normalized time $\bar{t}$, taking γ as a parameter for: (a) $\gamma = 0.015$, (b) $\gamma = 0.030$ and (c) $\gamma = 0.060$.

As shown in Figure 4, the higher the value of γ, which implies a lower value of the power supply voltage V_1, the longer the time required for the capacitor to fully charge, which occurs when $v_c = V_1$. This behavior is explained due to the nonlinearity caused by the voltage-dependent capacitance of the capacitor.

A. Charging Time

Substituting (24) and (25) in (22) and solving for $e^{\alpha t}$ we find

$$e^{\alpha t} = \frac{\left(\sqrt{(1+\gamma)\gamma} - \gamma - 1\right) - \left(\sqrt{(1+\gamma)} - \sqrt{\gamma}\right)\sqrt{\overline{v_c} + \gamma}}{\left(\sqrt{(1+\gamma)} + \sqrt{\gamma}\right)\sqrt{\overline{v_c} + \gamma} - \left(\sqrt{(1+\gamma)\gamma} + \gamma + 1\right)}. \quad (27)$$

Substitution of (23) in (19) yields

$$\alpha t = \frac{t}{RC_o}\sqrt{\frac{1+\gamma}{\gamma}}. \quad (28)$$

Substituting (25) in (28) we get

$$\alpha t = \bar{t}\sqrt{\frac{1+\gamma}{\gamma}}. \quad (29)$$

Substitution of (29) in (27) gives

$$e^{\bar{t}\sqrt{\frac{1+\gamma}{\gamma}}} = \frac{\left(\sqrt{(1+\gamma)\gamma} - \gamma - 1\right) - \left(\sqrt{(1+\gamma)} - \sqrt{\gamma}\right)\sqrt{\overline{v_c} + \gamma}}{\left(\sqrt{(1+\gamma)} + \sqrt{\gamma}\right)\sqrt{\overline{v_c} + \gamma} - \left(\sqrt{(1+\gamma)\gamma} + \gamma + 1\right)}. \quad (30)$$

Solving (30) for $\bar{t}$ we find

$$\bar{t}\left(\gamma, \overline{v_c}\right) = \sqrt{\frac{1+\gamma}{\gamma}} \ln\left[\frac{\left(\sqrt{(1+\gamma)\gamma} - \gamma - 1\right) - \left(\sqrt{(1+\gamma)} - \sqrt{\gamma}\right)\sqrt{\overline{v_c} + \gamma}}{\left(\sqrt{(1+\gamma)} + \sqrt{\gamma}\right)\sqrt{\overline{v_c} + \gamma} - \left(\sqrt{(1+\gamma)\gamma} + \gamma + 1\right)}\right]. \quad (31)$$

Equation (31) directly gives the normalized charging time $\bar{t}$ as a function of the normalized voltage $\overline{v_c}$ across the capacitor terminals, where γ is the parameter of the equation.

As follows from (26), the voltage across the capacitor asymptotically approaches the steady-state value, which is equal to the input voltage V_1, and the transient duration is theoretically infinite.

Let us define the rise time Δt_r as the time required for the capacitor voltage to rise from 10% to 90% of its final value, which is equal to V_1. Hence, the normalized voltage $\overline{v_c}$ rises from 0.1 to 0.9. The normalized rise time $\overline{\Delta t_r}$ is then

$$\overline{\Delta t_r}(\gamma) = \bar{t}(\gamma, 0.9) - \bar{t}(\gamma, 0.1). \quad (32)$$

Figure 5 presents the normalized rise time as function of γ, given by (32). As shown in this figure, the rise time increases with increasing γ, or with decreasing the voltage V_1.

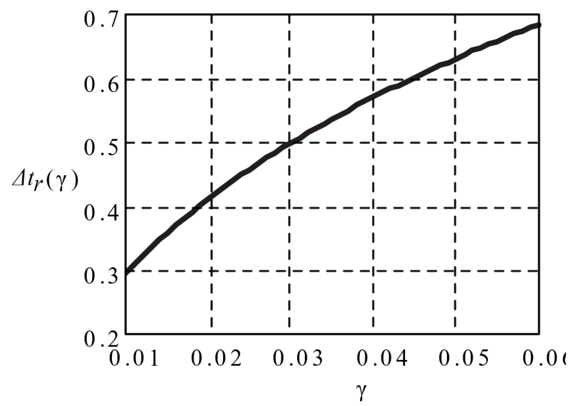

Fig. 5. Plot of the normalized rise time $\overline{\Delta t_r}$ versus γ.

B. Energy Stored in the Capacitor

The instantaneous power in the capacitor is

$$p_c = v_c i_c. \quad (33)$$

Substitution of (5) in (39) gives

$$p_c = C(v_c) v_c \frac{dv_c}{dt}. \quad (34)$$

The energy stored in the capacitor is given by

$$E_c = \int_t p_c dt = \int_t C(v_c) v_c \frac{dv_c}{dt} dt. \quad (35)$$

Hence

$$E_c = \int_{V_c} C(v_c)\, v_c\, dv_c. \tag{36}$$

Substitutions of (5) in (36) yields

$$E_c = \int_{V_c} \frac{C_o\sqrt{V_{bi}}}{\sqrt{V_{bi}+v_c}}\, v_c\, dv_c. \tag{37}$$

Integrating (37) in the interval $(0, V_1)$ we find

$$E_c = \int_{V_c} \frac{C_o\sqrt{V_{bi}}}{\sqrt{V_{bi}+v_c}}\, v_c\, dv_c. \tag{38}$$

C. Energy Supplied by the Voltage Source

The instantaneous power in the voltage source V_1 is

$$p_1 = V_1 i_c. \tag{39}$$

Substitution of (5) in (39) gives

$$p_1 = V_1 C(v_c)\frac{dv_c}{dt}. \tag{40}$$

The energy supplied by the voltage source is given by

$$E_1 = \int_t p_1\, dt = \int_t V_1 C(v_c)\frac{dv_c}{dt}\, dt. \tag{41}$$

Hence

$$E_1 = \int_{V_c} V_1 C(v_c)\, dv_c. \tag{42}$$

Substitutions of (5) in (42) yields

$$E_1 = \int_{V_c} V_1 \frac{C_o\sqrt{V_{bi}}}{\sqrt{V_{bi}+v_c}}\, dv_c. \tag{43}$$

Integrating (43) in the interval $(0, V_1)$ and rearranging the terms we find

$$E_1 = 2C_o V_1\left(\sqrt{V_{bi}}\sqrt{(V_1+V_{bi})} - V_{bi}\right). \tag{44}$$

Equation (44) gives the total energy transferred from the voltage source V_1 to the pair *RC* during the charging of the voltage-dependent capacitor.

D. Energy Dissipated in the Resistor

The energy E_R dissipated in the resistor R is the difference between the energy supplied by the voltage source V_1 and the energy stored in the capacitor, and is defined by

$$E_R = E_1 - E_c \tag{45}$$

Substitution of (38) and (44) in (45), after appropriate algebraic manipulation, yields

$$E_R = \frac{2}{3}C_o\sqrt{V_{bi}}\left(\sqrt{(V_1+V_{bi})} - \sqrt{V_{bi}}\right)\left[(2V_1+V_{bi}) - \sqrt{V_{bi}}\sqrt{(V_1+V_{bi})}\right]. \tag{46}$$

E. Ratio of Energy Dissipated in the Resistor and the Energy Stored in the Capacitor

Let us define λ as the ratio of the energy E_R dissipated in the resistor to the energy E_c stored in the capacitor. Thus,

$$\lambda = \frac{E_R}{E_c}. \tag{47}$$

Substituting (38) and (46) in (47) and rearranging the terms, we find

$$\lambda = \frac{2V_1 + V_{bi} - \sqrt{V_{bi}}\sqrt{V_1+V_{bi}}}{V_1 - V_{bi} + \sqrt{V_{bi}}\sqrt{V_1+V_{bi}}}. \tag{48}$$

Substitution of (23) in (48) yields

$$\lambda = \frac{2+\gamma-\sqrt{\gamma}\sqrt{1+\gamma}}{1-\gamma+\sqrt{\gamma}\sqrt{1+\gamma}}. \tag{49}$$

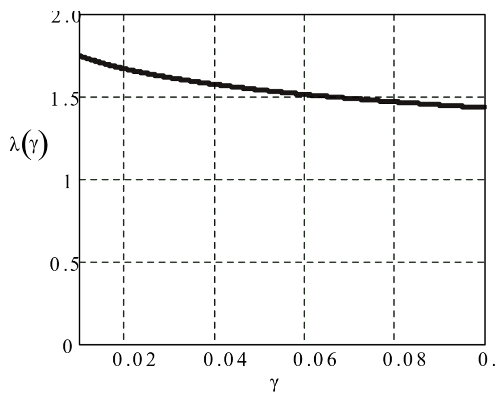

Fig. 6. Ratio of the energy dissipated in the resistor to the energy stored in the voltage-dependent capacitor, as function of γ.

According to equation (49), the ratio of the energy dissipated in the resistor R to the energy stored in the capacitor is independent of the values of R and C_o, being dependent only on the ratio of $\gamma = V_{bi}/V_1$.

A plot of λ versus , given by equation (49), is shown in Fig. 6, which shows that, in opposition of what occurs in the charging of a constant capacitance capacitor, the energy dissipated in the resistor is always larger than the energy stored in the capacitor. This phenomenon is attributed to the voltage-dependence of the capacitor.

IV. ANALYSIS OF THE DISCHARGING TRANSIENT OF THE VOLTAGE-DEPENDENT CAPACITOR

Since we are dealing with nonlinear systems, the principle of superposition will not hold. Therefore, the time dependence of charging voltage will differ from that of the corresponding discharge voltage, and the two cases must be considered separately.

The circuit that we shall consider is shown in Figure 7.

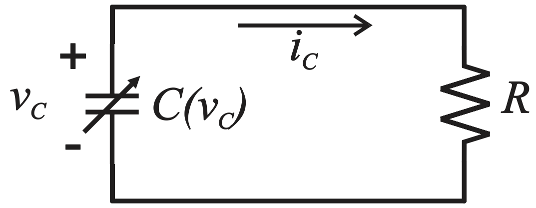

Fig. 7. Circuit for discharging the voltage-dependent capacitor through a linear resistor.

The current i_C is given by

$$i_C = -C(v_C)\frac{dv_C}{dt} = \frac{v_c}{R}. \quad (50)$$

Substitution of (5) in (50) yields

$$-\frac{C_o\sqrt{V_{bi}}}{\sqrt{V_{bi}+v_c}}\frac{dv_C}{dt} = \frac{v_C}{R}. \quad (51)$$

Hence,

$$-\frac{dv_C}{v_c\sqrt{V_{bi}+v_c}} = \frac{dt}{RC_o\sqrt{V_{bi}}} \quad (52)$$

and

$$-\int\frac{dv_C}{v_c\sqrt{V_{bi}+v_c}} = \int\frac{dt}{RC_o\sqrt{V_{bi}}}+K \quad (53)$$

where K is the constant of integration.

Integrating both terms of (53) we obtain

$$\frac{1}{\sqrt{V_{bi}}}\left[\ln\left(\frac{1}{\sqrt{V_{bi}+v_c}}+\frac{1}{\sqrt{V_{bi}}}\right)-\ln\left(\frac{1}{\sqrt{V_{bi}+v_c}}-\frac{1}{\sqrt{V_{bi}}}\right)\right]= \frac{t}{RC_o\sqrt{V_{bi}}}+K. \quad (54)$$

Making $t=0$ and $v_c(0)=V_o$ in (54) we obtain the constant of integration K, which is given by

$$K=\frac{1}{\sqrt{V_{bi}}}\left[\ln\left(\frac{1}{\sqrt{V_{bi}+V_o}}+\frac{1}{\sqrt{V_{bi}}}\right)-\ln\left(\frac{1}{\sqrt{V_{bi}+V_o}}-\frac{1}{\sqrt{V_{bi}}}\right)\right]. \quad (55)$$

Substituting (55) in (54) and rearranging the terms, we find

$$\ln\left(\frac{\dfrac{\sqrt{v_c+V_{bi}}+\sqrt{V_{bi}}}{\sqrt{v_c+V_{bi}}-\sqrt{V_{bi}}}}{\dfrac{\sqrt{V_o+V_{bi}}+\sqrt{V_{bi}}}{\sqrt{V_o+V_{bi}}-\sqrt{V_{bi}}}}\right)=\frac{t}{RC_o}. \quad (56)$$

Let us define the normalized quantities $\bar{t}$, γ_1 and $\overline{v_c}$ as

$$\bar{t}=\frac{t}{RC_o}. \quad (57)$$

$$\gamma_1=\frac{V_{bi}}{V_o} \quad (58)$$

and

$$\overline{v_c}=\frac{v_c}{V_o}. \quad (59)$$

Substitution of (57), (58) and (59) in (56) and appropriate algebraic manipulation yields

$$\overline{v_c}=\left(\frac{\left(\sqrt{\gamma_1}\sqrt{1+\gamma_1}-\gamma_1\right)+\left(\sqrt{\gamma_1}\sqrt{1+\gamma_1}+\gamma_1\right)e^{\bar{t}}}{\left(\sqrt{\gamma_1}-\sqrt{1+\gamma_1}\right)+\left(\sqrt{\gamma_1}+\sqrt{1+\gamma_1}\right)e^{\bar{t}}}\right)^2-\gamma_1 \quad (60)$$

where $\overline{v_c}$ is the normalized voltage across the voltage-dependent capacitor, during the discharge transient.

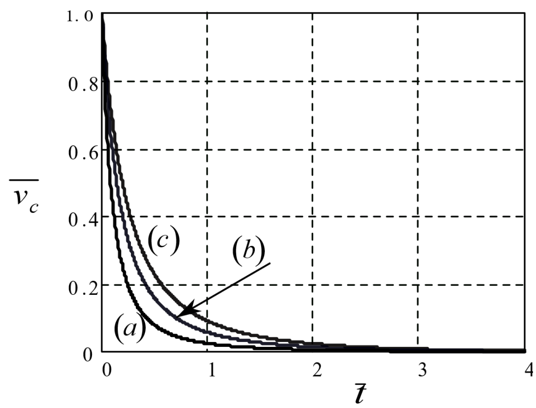

Fig. 8. Decay transient of the normalized voltage across the voltage-dependent capacitor versus normalized time, for: (a) $\gamma_1=0.010$, (b) $\gamma_2=0.030$ and (c) $\gamma_1=0.060$.

The plot of equation (60), as shown in Figure 8, indicates that the decay time of the voltage across the capacitor depends on the parameter $\gamma_1 = V_{bi}/V_o$. The higher the value of γ_1, which means lower value of the initial voltage V_o across the capacitor, the longer the decay time. This characteristic is due to the voltage-dependence of the capacitance. It should be noted that if the capacitance were independent of the capacitor voltage, the decay time would no longer be dependent on the capacitor initial voltage.

A. Discharging Time

From (60) we can write

$$\frac{\left(\sqrt{\gamma_1}\sqrt{1+\gamma_1}-\gamma_1\right)+\left(\sqrt{\gamma_1}\sqrt{1+\gamma_1}+\gamma_1\right)e^{\bar{t}}}{\left(\sqrt{\gamma_1}-\sqrt{1+\gamma_1}\right)+\left(\sqrt{\gamma_1}+\sqrt{1+\gamma_1}\right)e^{\bar{t}}}=\sqrt{\bar{v}_c+\gamma_1}. \quad (61)$$

We can find an explicit expression for the normalized time $\bar{t}_f$ by solving (61) and replacing $\bar{v}_c$ by $\left(1-\bar{v}_c\right)$ which yields

$$\bar{t}_f=\ln\left[\frac{\gamma_1+\left(\gamma_1-\sqrt{1+\gamma_1}\right)\sqrt{1-\bar{v}_c+\gamma_1}-\sqrt{\gamma_1}\sqrt{1+\gamma_1}}{\gamma_1+\gamma_1\sqrt{1+\gamma_1}-\sqrt{1-\bar{v}_c+\gamma_1}\left(\sqrt{\gamma_1}+\sqrt{1+\gamma_1}\right)}\right]. \quad (62)$$

Equation (62) directly gives the normalized discharge time $\bar{t}_f$ as a function of the normalized voltage across the capacitor terminals, where γ_1 is the parameter of the equation.

As follows from (62), the voltage $\bar{v}_c$ across the capacitor asymptotically approaches the steady-state value, which is equal to zero, and the transient duration is theoretically infinite.

Let us define the fall time Δt_f as the time required for the capacitor voltage to fall from 90% to 10% of its initial value, which is equal to V_1. Hence, the normalized voltage $\bar{v}_c$ falls from 0.9 to 0.1 . The normalized fall time $\overline{\Delta t_f}$ is then

$$\overline{\Delta t_f}\left(\gamma_1\right)=\bar{t}_f\left(\gamma_1,0.1\right)-\bar{t}_f\left(\gamma_1,0.9\right) \quad (63)$$

Curves computed from (63) for several values of γ_1 are presented in Figure 9.

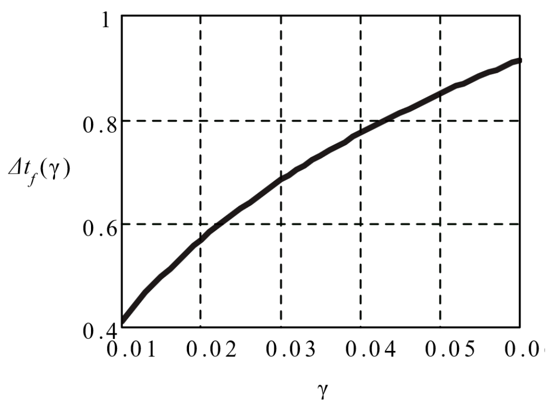

Fig. 9. Plot of the normalized fall time $\overline{\Delta t_f}$ versus γ_1.

B. Ratio Between the Normalized Rise and Fall Times

To compare the normalized times $\overline{\Delta t_r}$ and $\overline{\Delta t_f}$, both are plotted in Figure 10, where we note that the fall time is always larger than the rise time. These curves show clearly how the nonlinearity due to the voltage-dependence of the capacitance results in strong differences between charging and discharging times.

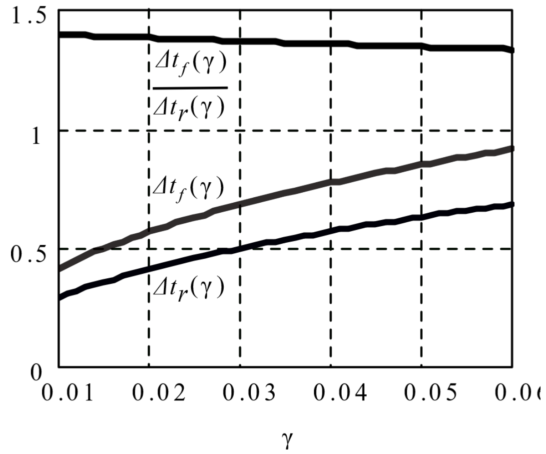

Fig. 10. Plot of the normalized fall times $\overline{\Delta t_f}$ and $\overline{\Delta t_r}$, and the ratio between them, versus γ .

V. EXPERIMENTAL RESULTS

In order to experimentally verify the validity of the theoretical analysis results, an experiment was carried out in the laboratory with the experimental circuit shown in Figure 11.

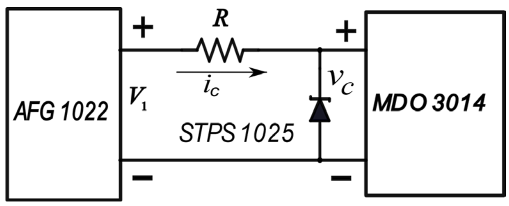

Fig. 11. Experimental RC circuit using the Schottky diode STPS10L25 as a voltage-dependent capacitor.

The Schottky diode STPS10L25 from STMicroelectroncis was used as the voltage-dependent capacitor, which was associated in series with a resistor . The parameters of the diode are $V_{bi}=0.326V$ and $C_o=4.5nF$ [7].

A rectangular voltage V_1 with appropriate frequency, amplitude and duty-cycle was generated from the Tektronix AFG 1022 function generator, which internal impedance is equal to 50Ω. The voltages v_C and V_1 were measured using the MDO 3014 Tektronix oscilloscope.

The measured charging voltages $v_{C\exp}$ and V_1 versus time, with $R=3k\Omega$ for $V_1=10V$ and $V_1=5V$, are shown in Figures 12 and 13, respectively, and the corresponding discharging voltages are plotted in Figures 14 and 15. In the same figures, the theoretical voltage across the capacitance of the diode v_{Ctheor} is represented by solid black symbols.

As figures show, the experimental and theoretical results are nearly identical and the differences can be attributed to the internal impedance of the function generator and the diode parasitic parameters, such as leakage current and series resistance, not included in the analysis.

It should be noted that, as predicted theoretically, the experimental charging and discharging times are sensitive to the value of the voltage V_1. The time interval for the capacitor voltage to reach $v_C = 0.9V_1$ is equal to $10\,\mu s$ when $V_1 = 10V$ and $16\,\mu s$ when $V_1 = 5V$. The discharging times are $6\,\mu s$ when $V_o = 10V$ and $8\,\mu s$ for $V_o = 5V$.

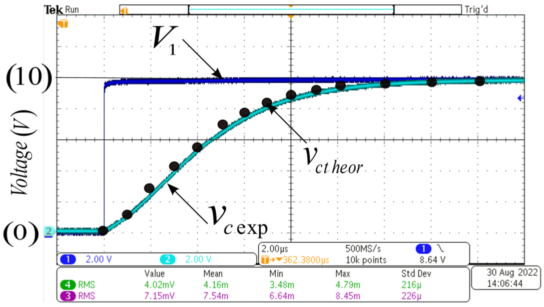

Fig. 12. Experimental results during the capacitor charging when $V_1 = 10V$. (a) Voltage V_1 generated by the function generator $(2V/div)$. (b) Experimental voltage $v_{c\exp}$ across the Schottky diode $(2V/div)$. (c) Theoretical voltage v_{ctheor} across the voltage-dependent capacitor $(2V/div)$.

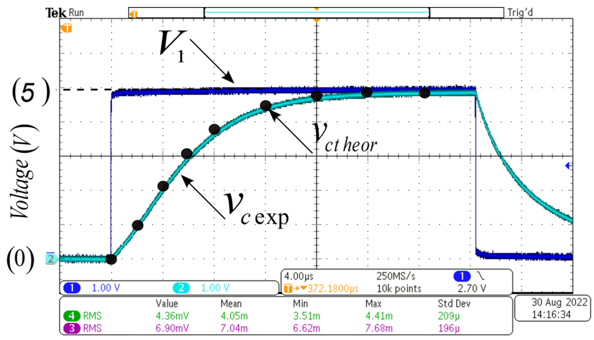

Fig. 13. Experimental results during the capacitor charging when $V_1 = 5V$. (a) Voltage V_1 generated by the function generator $(1V/div)$. (b) Experimental voltage $v_{c\exp}$ across the Schottky diode $(1V/div)$. (c) Theoretical voltage v_{ctheor} across the voltage-dependent capacitor $(1V/div)$.

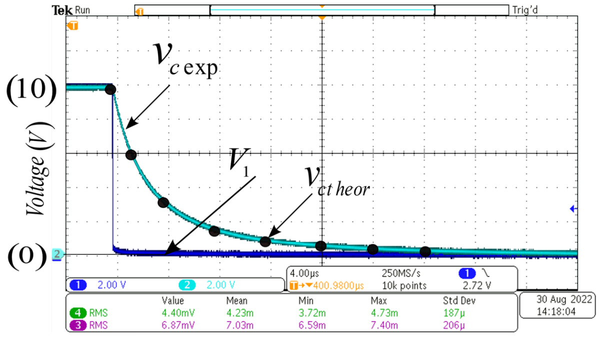

Fig. 14. Experimental results during the capacitor discharging when $V_1 = 10V$. (a) Voltage generated by the function generator $(2V/div)$. (b) Experimental voltage $v_{c\exp}$ across the Schottky diode $(2V/div)$. (c) Theoretical voltage v_{ctheor} across the voltage-dependent capacitor $(2V/div)$.

VI. CONCLUSIONS

In this paper we have presented theoretical and experimental study on the step function charging and discharging of Schottky diode nonlinear voltage-dependent capacitors, through a linear resistance. The circuits are described by first-order nonlinear differential equations, which closed form exact solution are presented.

From the performed study, we can draw the following conclusions: (a) the energy loss in the series resistor during charging is larger than the total energy transferred to and stored in the nonlinear capacitor, (b) the fall time is larger than the rise time, and (c) both the rise time and fall time are sensitive to the voltage value, decreasing with increasing voltage. These results indicate how the nonlinearity due to the voltage-dependence capacitances causes substantial differences between charging and discharging behavior.

A similar analysis can be extended to other voltage-dependent capacitors, such as ceramic capacitor, conventional P-N junction diodes capacitors and input and output capacitances of the MOSFET. The theoretical analysis results were verified by computer simulation and laboratory experimentation.

The results obtained in this study can be directly applied to determine the time intervals of the gate source and drain source voltages of power semiconductors during the commutation in static converters.

In the continuation of this research, parasitic inductances present in the switching loops of static converters will be included, which will allow the determination of the values of voltage peaks across the diodes at the moment they are turned off.

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BIOGRAPHY

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