D-TRANSFORM BASED CONTROL OF POWER CONVERTERS

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***Abstract* – The aim of this study is to present a very simple, intuitive and feasible tool: the D-transform between converters. This new method proposes to find a control equation of any pre-defined control law from one converter to another power converter. The central goal is to take maximum advantage of the robustness offered by the originator nonlinear control law. Although there is the possibility of more than one conversion, some candidates show good performance in terms of transient overshoot, settling time and steady-state regulation. The stability proof is disposed individually for each generated equation. The performance of D-Controllers is verified through Hardware In the Loop (HIL) simulations. A small overshoot under input and load perturbations is achieved for the buck-boost example. Finally, a experimental validation using a buck converter is given to illustrate the application method and the nonlinear control design.**

***Keywords* – Boost, Buck, Buck-Boost, D-Transform, HIL Simulation, IDAPBC, Nonlinear Control, Relations, SFL.**

NOMENCLATURE

E	Input voltage.
μ	Generalized duty cycle.
U	Steady-State duty cycle.
D	Boost duty cycle.
d	Buck duty cycle.
δ	Buck-boost duty cycle.
x_1	Inductor current.
x_2	Capacitor voltage.
V_d	Desired output voltage.
x_{2d}	Output voltage reference.
L	Converters inductance.
C	Converters capacitance.
G	Load conductance.
R	Load resistance.
x'	Complementer operator (1-x).
$\bar{x}$	Steady-state value.
k_1	SFL gain controller.
k_α	CIDAPBC gain controller.
k_z	IDAPBC gain controller.

I. INTRODUCTION

The design of non-conventional methods for solving power electronics and control problems is an emerging area [1], [2]. There are many reasons to use non-classical techniques, such as energy savings, cost reductions and, obviously, the increase of dynamic performance, large stability margin, and high robustness [3], [4], [5]. Therefore, this paper presents a new perspective regarding the control of static power converters that play a key role in several emerging applications including power systems [6], transformers [7], dc micro-grids [8] - [9], electric vehicles and aircrafts [10] - [11]], integrated energy systems [12], photovoltaic systems [13] l, cyber-physical systems [2], inverters [14], etc…

Through the employment of average nonlinear dynamic model, different nonlinear control methods have been addressed to regulate the desired voltage or current and achieve the stability of the closed-loop system [15]. Nonlinear controllers have been successfully applied to dc/dc converters established by a rigorous mathematical formulation, and are, in many cases, combined with the traditional PI control [16]. Nevertheless, almost all of the existing control methods for dc/dc converters requisite exact knowledge of the converter parameters (capacitance,inductance) or the load impedance to assurance nonlinear stability.

The Laplace, Fourier and Z-transforms [17] are remarkable tools in different domains, e.g., control, signal processing, telecommunications and electronic engineering. The central purpose is to directly apply the preferred and simplified equations that can be used in several practical applications. Inspired by this motion, we aim at designing a transform function of converter equations. To be specific, this work outlines a methodology to achieve nonlinear control based on D-Transform, which will be better described in dedicated section.

The first reason is the rapid generation of control equations extended of one converter to another. The second motivation is to produce a new family of controllers based on previously developed equations that will be redesigned. Such new control laws meet the requirements for new insights and robustness criteria of advanced controllers.

What is the D-Transform? By definition, the D-transform is comprised of finding, straightly, a function that converts an equation of the duty cycle μ from one converter to another converter, through the existing input to output relations of these converters, which are functions of the duty-cycle.

Moreover, the control methods used in this work are the State Feedback Linearization (SFL) [18] – [19], Interconnection and Damping Assignment Passivity-Based

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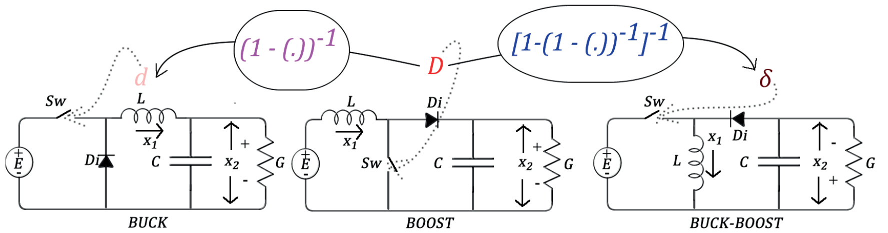

Fig. 1. Typical power converters and proposed D-Transform. Buck duty cycle (d), Boost duty cycle (D), and Buck-boost duty cycle(δ).

TABLE I
Converters Models

	Boost	Buck	Buck-Boost
SSM	$\dot{x}_1 = -(1-\mu)\frac{1}{L}x_2 + \frac{E}{L}$ $\dot{x}_2 = (1-\mu)\frac{1}{C}x_1 - \frac{G}{C}x_2$	$\dot{x}_1 = -\frac{1}{L}x_2 + \mu\frac{E}{L}$ $\dot{x}_2 = \frac{1}{C}x_1 - \frac{G}{C}x_2$	$\dot{x}_1 = (1-\mu)\frac{1}{L}x_2 + \mu\frac{E}{L}$ $\dot{x}_2 = -(1-\mu)\frac{1}{C}x_1 - \frac{G}{C}x_2$
SSM_n^0*	$\dot{x}_{1n} = -\mu' x_{2n} + 1$ $\dot{x}_{2n} = \mu x_{1n} - \frac{1}{Q}x_{2n}$	$\dot{x}_{1n} = -x_{2n} + \mu$ $\dot{x}_{2n} = x_{1n} - \frac{1}{Q}x_{2n}$	$\dot{x}_{1n} = \mu' x_{2n} + \mu$ $\dot{x}_{2n} = -\mu' x_{1n} - \frac{1}{Q}x_{2n}$
ELM	$D_B\dot{x} + (1-\mu)J_Bx + R_Bx = F$ $J_B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; D_B = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}; R_B = \begin{bmatrix} 0 & 0 \\ 0 & G \end{bmatrix}; F = \begin{bmatrix} E \\ 0 \end{bmatrix}$	$D_B\dot{x} + (J_B + R_B)x = \mu F$ $J_B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$	$D_B\dot{x} + (1-\mu)J_Bx + R_Bx = \mu F$ $J_B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix};$
PHC	$J_H = \begin{bmatrix} 0 & -\frac{1-\mu}{LC} \\ \frac{1-\mu}{LC} & 0 \end{bmatrix};$ $R_H = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{RC^2} \end{bmatrix}; g_H = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \dot{x} = [J_H(\mu) - R_H]\frac{\partial H}{\partial x}(x) + g_H E; H(x) = \frac{1}{2}Lx_1^2 + \frac{1}{2}Cx_2^2$	$J_H = \begin{bmatrix} 0 & -\frac{1}{LC} \\ \frac{1}{LC} & 0 \end{bmatrix};$ $R_H = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{RC^2} \end{bmatrix}; g_H = \begin{bmatrix} \frac{\mu}{L} \\ 0 \end{bmatrix};$	$J_H = \begin{bmatrix} 0 & \frac{1-\mu}{LC} \\ -\frac{1-\mu}{LC} & 0 \end{bmatrix};$ $R_H = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC^2} \end{bmatrix}; g_H = \begin{bmatrix} \frac{\mu}{L} \\ 0 \end{bmatrix};$

Control (IDA-PBC) [20] and other nonlinear control equations derived via D-Transform, whose models and control formulations are demonstrated in section II. The proposed D-Transform approach is explained in section III. Finally, results and conclusions are presented in section IV and V.

II. MODELING AND RELATIONS BETWEEN CONVERTERS

The basic dc-dc power converters - as boost, buck, and buck-boost shown in Figure 1 - are typical building blocks in power electronics, that have the same elements: one diode, one switch, one capacitor and one inductor. The difference between them comprises the physical position of such elements. Furthermore, the nonlinear behaviour is mainly present on semiconductor components (diode and switch).

Table I condenses three distinct models commonly found in the literature: Euler-Lagrange Model (ELM), State-Space Model (SSM) and Port-Controlled Hamiltonian (PCH). It should be emphasized that the Euler-Lagrange approach presents an equivalent form of buck-boost and boost modelling. In sequence, PCH model evidences a similar structure for the three converters. Possible problems encountered in boost and buck-boost converters are related to the occurrence of right-half-plane (RHP) zeros, which troublesome characteristic brings to nominimum-phase [21]. Additionally, RHP zero is the origin of bandwidth limitation and instability of closed-loop system [22].

normalized models are obtained by considering [23]: $\tau = \frac{t}{\sqrt{LC}}, x_{1n} = \frac{1}{E}\sqrt{\frac{L}{C}}x_1, x_{2n} = \frac{1}{E}x_2, \dot{x}_{in} = \frac{x_{in}}{d\tau}$

A. Equilibrium's Relation of Power Converters

The relations addressed in the following sentences - and summarized in Table II - are reported by the majority of power electronics book. Even so, we replicate some essential relations that will be employed to attain the proposed D-transform. It can be noted that these relations are for the ideal DC-DC converters in CCM (Continuous Current Mode) operation.

Boost

The circuit of boost converter circuit can be modelled by average space state equations:

$$\dot{x}_1 = -(1-D)\frac{1}{L}x_2 + \frac{E}{L}; \dot{x}_2 = (1-D)\frac{1}{C}x_1 - \frac{G}{C}x_2. \quad (1)$$

where x_1 denotes the inductor current, x_2 is the capacitor voltage, E is the input voltage, x_n is the normalized state variable.

In the steady state, by substituting $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ in (1), the equilibrium points of the boost converter are given by:

$$\bar{x}_1 = \frac{EG}{(1-D)^2}, \bar{x}_2 = \frac{E}{(1-D)}. \quad (2)$$

By considering $\bar{D}$ as a constant control input in view of (2), one gets:

$$\bar{x}_1 = \frac{G}{E}\bar{x}_2^2. \quad (3)$$

Now, replacing the desired output capacitor voltage as $\bar{x}_2 = V_d$, the equilibrium points $\bar{x}$ and the fixed input control

TABLE II
Relation Equations

	SFL control equation	Open loop ($U=\bar{d},\bar{D},\bar{\delta}$)		Slew rate	$\frac{V_d}{E}$		Transformation
Buck	$d=\frac{-k_1(x_1-x_{1d})+x_2}{E}$	$\bar{d}=1-\frac{E-V_d}{E}$	$\bar{d}=\frac{V_d}{E}$	$\bar{d}'=1-\bar{d}=\frac{E-V_d}{E}$	μ	d	$d=(D')^{-1}$
Boost	$D=1-\frac{\left[E+k_1(x_1-x_{1d})\right]}{x_2}$	$\bar{D}=1-\frac{E}{V_d}$	$\bar{D}=\frac{V_d-E}{V_d}$	$\bar{D}'=1-\bar{D}=\frac{E}{V_d}$	$\frac{1}{(1-\mu)}$	$\frac{1}{(1-D)}$	
Buck-Boost	$\delta=\frac{+k_1(x_1-x_{1d})+x_2}{x_2-E}$	$\bar{\delta}=1-\frac{E}{E-V_d}$	$\delta=\frac{V_d}{E-V_d}$	$\bar{\delta}'=1-\bar{\delta}=\frac{E}{E-V_d}$	$-\frac{\mu}{(1-\mu)}$	$-\frac{\delta}{(1-\delta)}$	$\delta=1-(d')^{-1}$

TABLE III
Transfer Functions (Found in [24])

Converter	G_{i0}	G_{d0}	w_0	Q	w_z
Buck	U	$\frac{V_d}{U}$	$\frac{1}{\sqrt{LC}}$	$R\sqrt{\frac{C}{L}}$	∞
Boost	$\frac{1}{U'}$	$\frac{V_d}{U'}$	$\frac{U'}{\sqrt{LC}}$	$U'R\sqrt{\frac{C}{L}}$	$\frac{U'2R}{L}$
Buck-boost	$\frac{-U}{U'}$	$\frac{V_d}{UU'}$	$\frac{U'}{\sqrt{LC}}$	$U'R\sqrt{\frac{C}{L}}$	$\frac{U'2R}{UL}$

$\bar{D}$ are given by:

$$\bar{D}=1-\frac{E}{V_d},\bar{x}=[\bar{x_1},\bar{x_2}]^T=\left[GV_d\left(\frac{V_d}{E}\right),V_d\right]^T. \tag{4}$$

Buck

Next let us consider the buck converter, some simple calculations show that:

$$\begin{aligned}\bar{x}_1=dEG,\bar{x}_2=dE,\bar{x}_1=G\bar{x}_2,\bar{d}=1-\frac{E-V_d}{E},\\ \bar{x}=[\bar{x}_1,\bar{x}_2]^T=[GV_d,V_d]^T.\end{aligned} \tag{5}$$

The State-Space Modelling of step down converter is given by:

$$\dot{x}_1=-\frac{1}{L}x_2+d\frac{E}{L}, \tag{6}$$

$$\dot{x}_2=\frac{1}{C}x_1-\frac{G}{C}x_2, \tag{7}$$

The equilibrium points of buck converter is obtained when replacing $\dot{x}=0$ in (6)-(7):

$$\bar{x}_1=dEG, \tag{8}$$

$$\bar{x}_2=dE. \tag{9}$$

By considering d as a fixed value in (8)-(9) leads to:

$$\bar{x}_1=G\bar{x}_2. \tag{10}$$

Therefore, the fixed open loop control $\bar{d}$ to stabilize $\bar{x}$ is:

$$\begin{aligned}\bar{d}=1-\frac{E-V_d}{E},\\ \bar{x}=[\bar{x}_1,\bar{x}_2]^T=[GV_d,V_d]^T.\end{aligned} \tag{11}$$

Buck-boost

The equivalent SSM of buck-boost can be expressed in the following form:

$$\dot{x}_1=(1-\delta)\frac{1}{L}x_2+\delta\frac{E}{L}, \tag{12}$$

$$\dot{x}_2=-(1-\delta)\frac{1}{C}x_1-\frac{G}{C}x_2, \tag{13}$$

When substituting $\dot{x}_1=0$ and $\dot{x}_2=0$ in (12)-(13), one obtains:

$$\bar{x}_1=\frac{\delta EG}{(1-\delta)^2}. \tag{14}$$

$$\bar{x}_2=-\frac{\delta E}{(1-\delta)}. \tag{15}$$

By replacing $\bar{\delta}$ in (14)-(15), we have:

$$\bar{x}_1=G\bar{x}_2\left(\frac{\bar{x}_2}{E}-1\right). \tag{16}$$

Thus, the equilibrium points to stabilize $\bar{x}$ and the constant input control $\bar{\delta}$ given by:

$$\begin{aligned}\bar{\delta}=1-\frac{E}{E-V_d},\\ \bar{x}=[\bar{x}_1,\bar{x}_2]^T=\left[GV_d\left(\frac{V_d}{E}-1\right),V_d\right]^T.\end{aligned} \tag{17}$$

So, the summarized equations for buck-boost converter are:

$$\begin{aligned}\bar{x}_1=\frac{\delta EG}{(1-\delta)^2},\bar{x}_2=-\frac{\delta E}{(1-\delta)},\bar{x}_1=G\bar{x}_2\left(\frac{\bar{x}_2}{E}-1\right),\\ \bar{\delta}=1-\frac{E}{E-V_d},\bar{x}=[\bar{x}_1,\bar{x}_2]^T=\left[GV_d\left(\frac{V_d}{E}-1\right),V_d\right]^T.\end{aligned} \tag{18}$$

B. Transfer Functions of Linearized Models

Table III exhibits parameters of both control-to-output (G_{vd}) and input-to-output (G_{vi}) transfer functions of the basic boost, buck and buck-boost converters [24]:

$$G_{vi}(s)=G_{i0}\frac{1}{1+\frac{s}{Qw_0}+(\frac{s}{w_0})^2}, \tag{19}$$

TABLE IV
Canonical Circuit Parameter (Found in [24])

Converter	$M(U)$	L_e	$e(s)$	$j(s)$
Buck	U	L	$\frac{V_d}{U^2}$	$\frac{V_d}{R}$
Boost	$\frac{1}{U'}$	$\frac{L}{U'^2}$	$V_d\left(1-\frac{sL}{U'^2R}\right)$	$\frac{V_d}{U'^2R}$
Buck-boost	$\frac{-U}{U'}$	$\frac{L}{U'^2}$	$\frac{-V_d}{U^2}\left(1-\frac{sL}{U'^2R}\right)$	$\frac{-V_d}{U'^2R}$

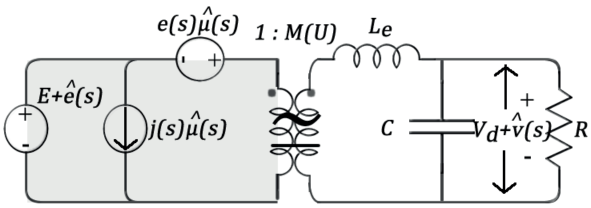

Fig. 2. Canonical power converters circuits.

$$G_{vd}(s) = G_{d0}\frac{\left(1-\frac{s}{w_z}\right)}{\left(1+\frac{s}{Qw_0}+\left(\frac{s}{w_0}\right)^2\right)}. \qquad (20)$$

The canonical form of converters is shown in Figure 2. The straight derivation of the transfer functions is displayed in Table IV. All tables presented in this work make an effort to clarify common characteristics among the converts, so they are customized as potential sources of D-candidates.

III. MAIN IDEA: THE D-TRANSFORM

Figure 3 shows the flowchart of the proposed methodology. The D-Transform design proceeds in following steps:

1. Converter: choose a converter as a starting point. In this work, the boost was selected.
2. Inputs: it is possible to apply any nonlinear control based equations. We use SFL and IDAPBC control laws, for example.
3. D-Transform: the next step is to apply the D-transform, firstly, by considering intuitive relations (e.g.: steady-state equations), and then nontrivial relations.
4. Outputs: the new D-Controllers (e.g.: D-SFL, D-CIDAPBC, D-IDAPBC).
5. Validation: finally, the new generated control equations can be verified by software simulations, stability analysis and hardware tests.

Definition 1. *The D-transform consists of finding, directly, a function that establishes the duty cycle μ from one converter to another.*

Surrounded by several candidates, we choose as initial point the following transformation equations:

Proposition 1. *There is a transformation function Δ_1 that converts $D \rightarrow d, \delta$:*

$$d, \delta = \Delta_1(D), d = (D')^{-1}, \delta = 1 - \left[1-(D')^{-1}\right]^{-1}. \qquad (21)$$

Example 1. *The root of Δ_1 is relationed to steady-state investigation. To simplify the notation, let us replicate the buck and boost equations given by* (4)-(5)*:*

$$\bar{D} = \frac{V_d - E}{V_d}, \bar{d} = \frac{V_d}{E}. \qquad (22)$$

It is possible to define a expression that leads $\bar{D} \Rightarrow \bar{d}$ (consequently, D $\Rightarrow d$). It should be noted that the equation conversion is given by (21) and the second column of Table III.

Proposition 2. *There exist other transformation functions $\Delta_2, \Delta_3, \ldots, \Delta_n$ that convert $D \rightarrow \delta$:*

$$\delta = \Delta_2(D), \delta = -\frac{D^{-1}}{D'} \qquad (23)$$

$$\delta = \Delta_3(D), \delta = 1 + D'. \qquad (24)$$

Example 2. *Let us repeat the control gain G_{io} of Table III:*

$$G_{io} = \frac{-U}{U'}, \qquad (25)$$

By replacing U - generalized duty cycle in steady-state - by D it is important to recall that (23) and (25) are similars.

To investigate the applications of D-transform we choose recently nonlinear equations found in the literature. For simplicity, we use two control laws based on IDAPBC. In [25], Classic IDAPBC (referenced as CIDAPBC) is addressed to boost converters achieving a simple control equation described by: *CIDAPBC*:

$$\bar{D} = 1 - \frac{E}{V_d}, D = 1 - \left(1-\bar{D}\right)\left(\frac{x_2}{V_d}\right)^{k_\alpha}. \qquad (26)$$

In addition, [20] modify and improve (26) to obtain:
IDAPBC:

$$D = 1 - \frac{k_z E}{2Ex_2 + (k_z - 2E)x_{2d}}. \qquad (27)$$

For didactic purposes, we also add the nonlinear SFL equation [15]:

$$D = 1 - \frac{[E + k_1(x_1 - x_{1d})]}{x_2}. \qquad (28)$$

By replacing the results of (26)-(28) in (21)-(24), we collect the new D-equations gathered in Table II (where k_1, k_α and k_z are the nonlinear control gains).

Example 3. *So, the new D-SFL control equation is performed by evaluation of* (21) *and* (28)*:*

$$d = \frac{x_2}{[E + k_1(x_1 - x_{1d})]}. \qquad (29)$$

Example 4. *By substituting* (27) *in* (24)*:*

$$\delta = 1 + \frac{k_z E}{2Ex_2 + (k_z - 2E)x_{2d}}. \qquad (30)$$

A. Stability Analyses

Two main goals are to be analyzed, regarding the stability of the closed-loop system: (i) the equilibrium of the system and (ii) the zero dynamics at the equilibrium. The control design consists in, first, rendering the error $(x - x_d)$ equal to zero, and, there, ensuring asymptotic stability to the error dynamics.

TABLE V
D-Transform Control Equations

	SFL	D-SFL		IDAPBC	D-IDAPBC
Buck	$d = \frac{-k_1(x_1-x_{1d})+x_2}{E}$	$d = \frac{x_2}{E+k_1(x_1-x_{1d})}$	$\leftarrow$ **Eq.**(21) $\rightarrow$	$\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow$	$d = \frac{2Ex_2+(k-2E)x_{2d}}{kE}$
Boost (source)	$D = 1 - \frac{[E+k_1(x_1-x_{1d})]}{x_2}$	-		$D' = \frac{kE}{2Ex_2+(k-2E)x_{2d}}$	-
Buck-Boost	$\delta = \frac{+k_1(x_1-x_{1d})+x_2}{x_2-E}$	$\delta = \frac{-x_2}{k_1(x_1-x_{1d})+E-x_2}$	$\leftarrow$ Eq.(21)	Eq.(24) $\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow$	$\delta = 1 + \frac{kE}{2Ex_2+(k-2E)x_{2d}}$

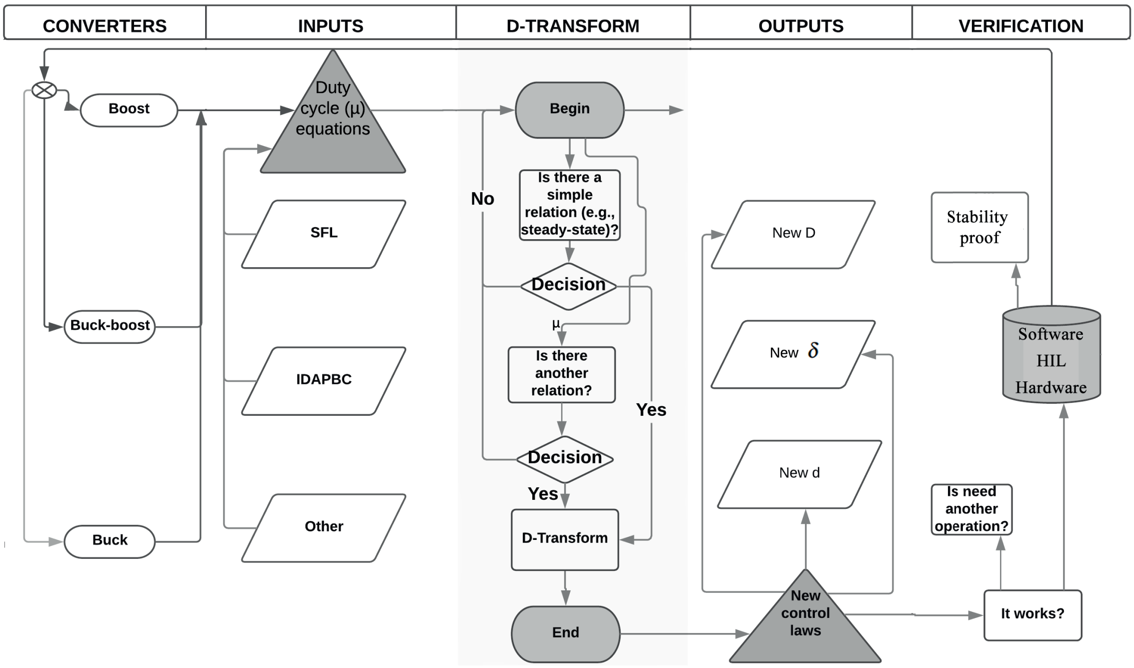

Fig. 3. Flowchart of D-Transform.

TABLE VI
D-Transform Control Equations

	CIDAPBC	D-CIDAPBC
Buck	-	$d = 1-\left(1-\bar{d}\right)\left(\frac{x_2}{V_d}\right)^{-k\alpha}$
Boost	$D = 1-\left(1-\bar{D}\right)\left(\frac{x_2}{V_d}\right)^{k\alpha}$	
Buck-Boost	-	$\delta = 1-\left(1-\bar{\delta}\right)\left(\frac{x_2}{V_d}\right)^{k\alpha}$

As proved by [26], the system is asymptotically stable if the new control law is satisfied and the ‘zero dynamics’ around the desired equilibrium point are stable. In order to evaluate the stability of the internal dynamics of the closed loop system, the standard approach is to consider the corresponding zero dynamics. As long as the zero dynamics is asymptotically stable, the internal dynamics will be locally exponentially stable [27]. Thus, the equation describing the zero-order dynamics is obtained by making the error equal to zero and replacing the state variables by their respective steady-state values.

Example 5. *The zero-order dynamics of the D-SFL buck control equation is obtained by using terms of* (5) *and* (29)*:*

$$\bar{\mu} = \frac{x_{20}}{E}, \tag{31}$$

$$\dot{x}_{20} = \frac{\bar{x}_1 - G\bar{x}_2}{C}. \tag{32}$$

As $\bar{x}_2 = \bar{\mu}E$ *and* $\bar{d} = \bar{\mu}$ *for the buck converter, deriving the two sides of* (31) *and substituting in* (32)*, the zero order dynamics as a function of* $\bar{\mu}$*:*

$$\dot{\bar{\mu}} = \frac{G}{C}\left(\bar{\mu} - \frac{\bar{x}_1}{GE}\right) \tag{33}$$

which the equilibrium point, $\bar{\mu} = \frac{\bar{x}_1}{GE}$*, is stable (similar procedure is obtained in [26] and [28]).*

B. D-Transform and δ*-Transform*

We choose the boost as base converter to generate D-Controlers for buck and buck-boost converters. However, the same strategy can be applied to attain the new D control laws from the buck (d-Transform) and also from the buck-boost (δ-Transform). In other to clarify this suggestion, the evaluation of $d \rightarrow \delta$ is given by:

$$\delta = 1 - (d')^{-1}. \tag{34}$$

The D-Controllers achieved for the buck-boost can be adapted to flyback converter, which is widely used in cellular power supplies and in other applications like LED and solar power

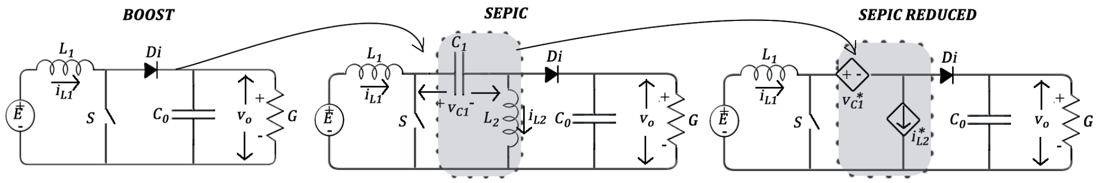

Fig. 4. Reduced SEPIC

system, for example [29]. For generation equations from buck to boost converter:

$$D = \left(d^{-1}\right)'. \tag{35}$$

Example 6. *Let us replicate the buck and boost equations given by* (4)-(5)*:*

$$\bar{D} = \frac{V_d - E}{V_d}, \bar{d} = \frac{V_d}{E}. \tag{36}$$

It is possible to define a expression that leads $\bar{d} \Rightarrow \bar{D}$ (consequently, $d \Rightarrow D$). It should be noted that the equation conversion is given by (35).

C. Extension to Other Converters

Figure 4 shows the boost and SEPIC converters for comparison. It can be noted that when removing the intermediate elements (L_2, C_1) highlighted by the dotted line, the SEPIC converter becomes similar to the boost converter, having similar equilibrium points as shown in Table VII.

Therefore, we can use the boost equations and apply them to control the SEPIC converter replacing the eliminated state variables (i_{L2} and v_{c1}) by the equilibrium points, provided in Table VII, represented by dependent sources in the model. If we substitute, for example, v_{c1} by E and i_{L2} by GV_d in the SEPIC converter, it saves two sensors. Figure 4 summarizes this process. So the equation to transform the boost duty cycle D to SEPIC duty cycle Ds is:

$$D = Ds. \tag{37}$$

For CUK converter, Equation (37) can be applied as reported in [30].

D. The Integral Action

The transformations, the nonlinear models and control equations require accurate knowledge of the converter parameters. Thus, in order to constrain the voltage output to get the desired value V_d, it is useful to include a proportional integrative (PI) term in the control equation, given by [31]:

$$G_{Int} = -k_{int} \int_0^t [x_2(s) - V_d] ds. \tag{38}$$

TABLE VII
Equilibrium Points of the Converters

Converter	i^*_{L1}	v^*_o	i^*_{L2}	v^*_{C1}
Boost	$\frac{G}{E}V_d^2$	V_d	-	-
SEPIC	$\frac{G}{E}V_d^2$	V_d	GV_d	E

Equation (38) can be applied for all converters using D-Controlers, IDAPBC, SFL or other nonlinear control law. For example, by considering the D-SFL control and the buck converter:

$$d = \frac{x_{2d}}{[E + k_1(x_1 - x_{1d})]},$$

$$x_{2d} = -k_{int} \int_0^t [x_2(s) - V_d] ds. \tag{39}$$

IV. SOFTWARE SIMULATION AND HIL RESULTS

Simulations are made to compare the performance of D-Controllers using Matlab/Simulink and Single Hardware in The Loop (SHIL) approach [32]. In all simulations, we have chosen the system parameters and design specifications shown in Table 7 and the D-control laws (Tables V and VI).

In Figure 5.b we present the transient responses of the output voltage of the buck-boost under load perturbation, by considering original SFL and derived D-SFL, D-IDAPBC and D-CIDAPBC control methods. The same scheme is shown in Figure 5.c for the buck converter. In these simulations, consecutive load step variation of 70% to 100% are applied. Figure 5.a, D shows the output capacitor voltage x_2 using Hardware in the loop. Figure 6 reveals the input variation tests.

As seen in Figures 4 and 5, both software simulations and HIL results reach the steady state value after the step variation. It is remarkable that the performance of the systems are in agreement, since the same transient dynamics is observed, even for the oscillatory dynamics. None of the implemented systems in software or in HIL presents instability. This attests that the embedded models and control equations are adequate to controlling the systems, and so, validating the control techniques. The processing time for the DSP (model Texas Instruments 28389D) to evaluate the control law and, consequently, to run a real-time simulation, is 1.2 μs. Using a switching frequency $f_s = 50kHz$ ($T_s = 20\mu$s), the processing time of all control equations ($<1.5\mu$s) demands 6% of the bandwidth.

V. EXPERIMENTAL VALIDATION

The proposed control strategy was experimentally validated through a DC-DC converter prototype, Figure 12, development with kit BOOSTXL-3PHGANINV Evaluation Module of Texas Intruments [33]. The prototype consists of a three-phase GaN bridge module and a module of inductors and capacitors forming the passive elements of the buck and boost.

Figure 7 (PI) and Figure 8 (D-SFL) show the output voltage

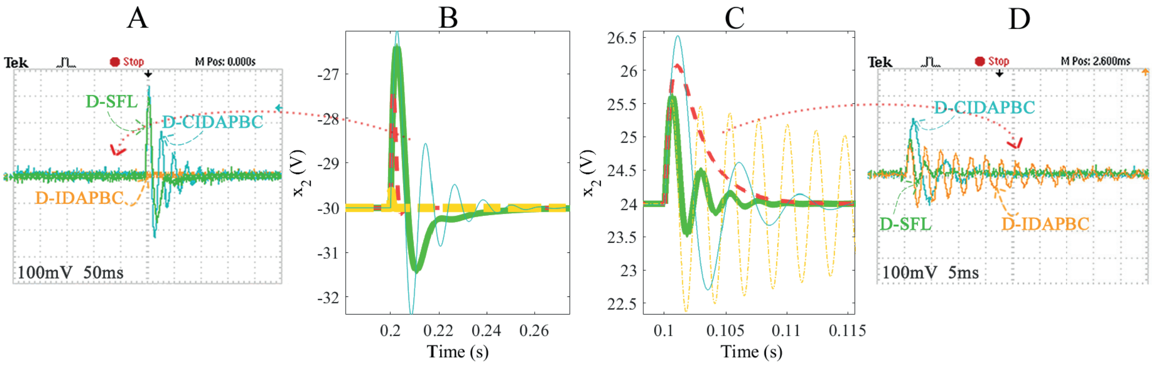

Fig. 5. Software simulation result (Matlab/Simulink) using control techniques SFL (dashed red), D-SFL (continue green) D-CICAPBC (continue cyan), D-IDAPBC (dashed yellow). Output voltage x_2 for buck-boost (B) and buck (C) in view of load variation (70-100 %) in 0.1s and 0.2s.HIL experimental result, normalized output voltage x2 (A-D).

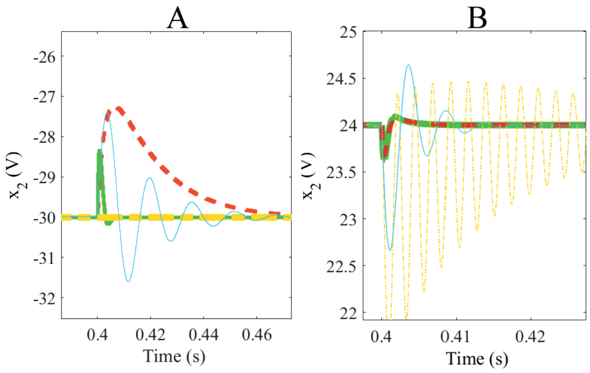

Fig. 6. Software simulation result (Matlab/Simulink) using control techniques SFL (dashed red), D-SFL (continue green) D-CICAPBC (continue cyan), D-IDAPBC (dashed yellow). Output voltage x_2 for buck-boost (A) and buck (B) in view of input variation (50 $\rightarrow$ 30 V) in 0.4s.

and the inductor current for a step variation of setpoint $x_{1d} = 1A \Rightarrow 3$ A. The output reference voltage V_d is set initially to 4 V, at t = 2 ms it changes to 12 V and at t = 14 ms returns to 4 V. Figure 9 presents the details of experimental waveforms. As it is shown in Figures 10 and 11, during the first 10 ms, both PI and D-SFL controllers regulate the output voltage at the desired level after transient.

The regulator design is typically driven by specifications concerning the required closed loop speed of response or, equivalently, the maximum allowed tracking error with respect to the reference signal. These specifications can be turned into

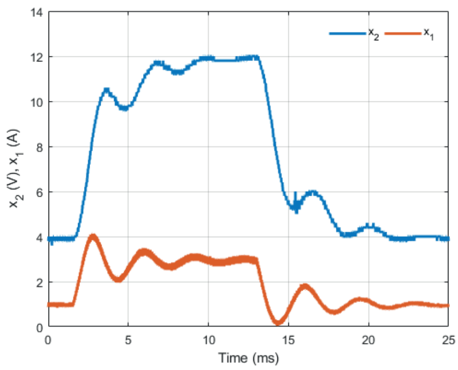

Fig. 7. Experimental result for buck - PI. Output voltage and inductor current - Setpoint variation.

TABLE VIII
Initial and Converters Parameters

Parameters	Buck-boost	Buck
x_{1d}	$\frac{G}{E}V_d^2$	GV_d
R	10 Ω	10 Ω
L	322 μh	322 μH
C	400μF	400μF
E	50 V	50 V
V_d	-30 V	25 V
f	50 kHz	50 kHz
k_1	3	3
k_z	-300	-500
k_{int}	-300	-300
k_α	0,77	0,77

TABLE IX
Experimental Buck

x_{1d}	$\frac{G}{E}V_d^2 = 1$A
R	4 Ω
L	322 μH
C	400μF
E	24 V
V_d	12 V
f	20 kHz
k_1	6
k_{int}	5

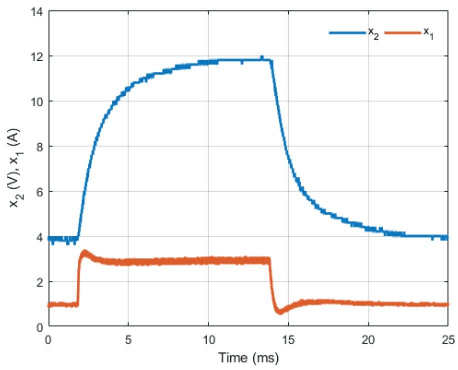

Fig. 8. Experimental result for buck - D-SFL. Output voltage and inductor current- Setpoint variation.

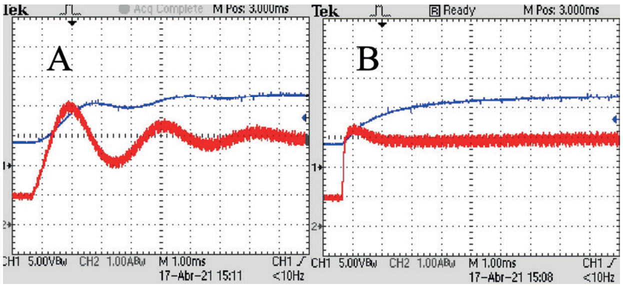

Fig. 9. Details of experimental results for buck - PI (A) and D-SFL (B). Output voltage (blue) and inductor current (red) - 12 V Setpoint.

equivalent specifications for the closed loop bandwidth and phase margin [34]. In our case, the current controller, a closed loop bandwidth equal to about 1/5 of the switching frequency (fc = 20 kHz), to be achieved with, at least, a 60 degrees phase margin.

In order to test the d-transform property of the controllers, the value of input voltage E is changed (24V $\Rightarrow$ 12V $\Rightarrow$ 24V) as shown in Figures 10 and 11. It is possible to verified that the D-SFL response presents less oscillations and smaller overshoot. Thus, the phase margin is greater for D-SFL compared with the classical PI.

VI. CONCLUSION

A new family of nonlinear controllers was proposed using the D-transform. In a new way, this method has a potentially useful properties for power converter applications. Based on the boost converter, control laws were generated for the buck and buck-boost converters (new d and new δ). Different alternatives were presented to show more than one way to find other candidates, which are proved to be stable control laws and eventually to present better performance.

The main contribution of this paper is to open new possibilities of stable nonlinear control laws for dc-dc converters, by a simple and direct methodology. It should be noted that the result obtained for the buck-boost – shown in Figures 5.a and 6.a (yellow line) – is quite feasible and contains the following advantages:

- low overshoot under load and input variations;
- rapid response speed;
- does not require the measurement of the inductor current x_1 simplifying the controller design;
- low number of counts (compared with the work of [35]).

Future research will concentrate in generalized stability proof of the transformations and the application extension to SEPIC, CUK and Three-phase converters. Additionally, the inverse process of generating equations having the buck and buck-boost as base converters will be investigated.

ACKNOWLEDGEMENT

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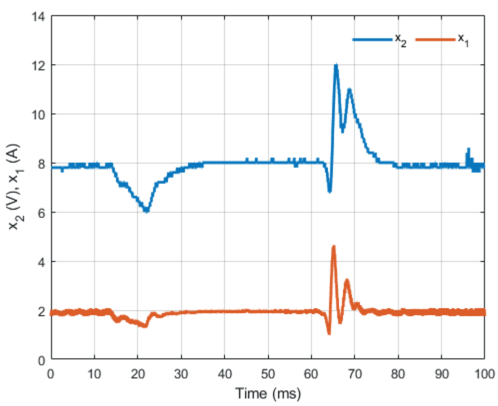

Fig. 10. Experimental result for buck - PI. Output voltage and inductor current with change of input voltage.

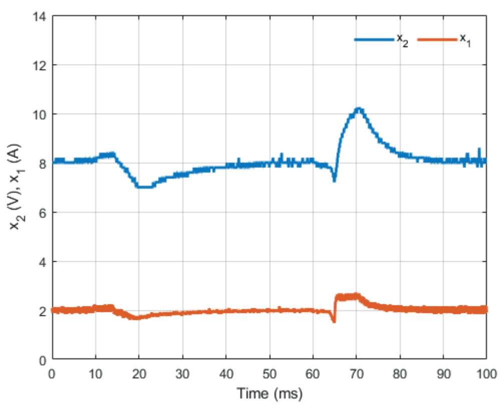

Fig. 11. Experimental result for buck - D-SFL. Output voltage and inductor current with change of input voltage.

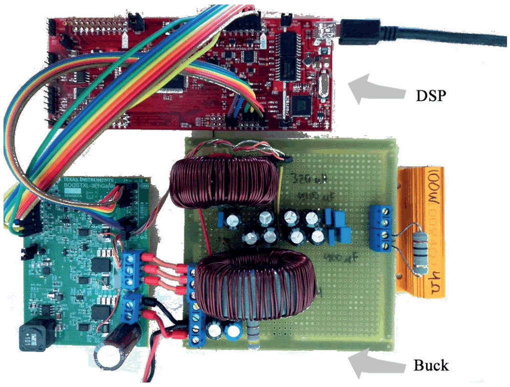

Fig. 12. DC-DC converter prototype

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